4. Mathematical model for robot dynamics
Part I

Dynamic Systems

- Systems that are not static, i.e., their state evolves with respect to time, due to:
  - input signals
  - external perturbations
  - or naturally
- For example, a dynamic system is a system which changes:
  - its trajectory $\longrightarrow$ changes in acceleration, orientation, velocity, position
  - its current, voltage, frequency, etc.
  - its temperature, pressure, etc.

Examples of dynamic systems

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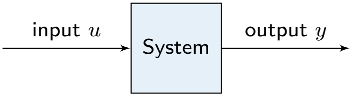

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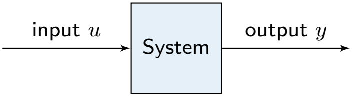

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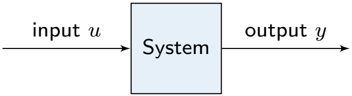

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- Model-based system description: based on physical first principles. Compared to experimental methods, it has 2 major benefits. The models obtained:
  - we can extrapolate the behavior of the system (valid beyond the operating conditions used in model validation)
  - it is useful when the real system is not available (because it is dangerous or not possible to test it on real systems or the system is still in planning phase)

(Translational) Mechanical systems

Variables: the symbols for the basic variables used to describe the dynamic behavior of translational mechanical systems

- x, displacement in meters (m)
- v, in meters per second (m/s)
- a, acceleration in meters per second per second (m/s^2)
- f, force in newtons (N)
- All these variables are functions of time. Note the equations:

$$v = \frac{dx}{dt} \qquad\qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Element laws: systems are represented by one or more idealized elements that obey laws involving the variables associated with the elements

- A mass M satisfies Newton's second law: $f = Ma$
- A mass M sliding on a viscous fluid experiences friction force f and satisfies $f = Bv$, where B is constant and depends on the fluid
- Any mechanical element (e.g., spring) that undergoes a change in shape, when subjected to a force, satisfies $f = Kx$, where K is a stiffness constant