Overview: Collective Control of Multi-agent Systems

Steffi Knorn, Zhiyong Chen, and Richard Middleton

***Abstract*—Collective control of a multi-agent system is concerned with designing strategies for a group of autonomous agents operating in a networked environment. The aim is to achieve a global control objective through distributed sensing, communication, computing, and control. It has attracted many researchers from a wide range of disciplines, including the literature of automatic control. The present paper aims to give a general framework that is able to accommodate many of these outcomes. Within this framework, the development on this topic is systematically reviewed and the representative outcomes can be sorted out from four aspects: (i) agent dynamics, (ii) network topologies, (iii) feedback and communication mechanisms, and (iv) collective behaviors. Thus, the state-of-the-art approach and technology is described. Moreover, within this framework, further interesting and promising directions on this research topic are envisioned.**

***Index Terms*—Multi-agent systems, networked systems, collective control, collaborative control, autonomous agents**

I. Introduction

A multi-agent system refers to a group of autonomous agents operating in a networked environment. Control engineers are interested in designing strategies for a multi-agent system to achieve certain global control objective through distributed sensing, communication, computing, and control. Small multi-agent systems are exemplified by formation of unmanned aircraft vehicles and cooperative microrobots. Large multi-agent systems include smart grid, traffic networks, sensor networks, biological systems and social networks. Common global control objectives include consensus, synchronization, formation, etc.

The most important feature of multi-agent systems is their autonomous nature, which demands distributed operations. It is this feature that allows a multi-agent system to be scalable in the sense that, when the network size increases, similar global control objectives can still be achieved without increasing the complexities for sensing, communication, computing and control. In order to achieve the required global control objective, a communication network is essential for agents to share information among their neighbors.

Collective control of multi-agent systems has attracted a great deal of research over recent decades. Surveys on the topic include [1], [2], [3], [4] and books include [5], [6], [7], [8], [9], [10], [11], [12]. The present paper aims to systematically review these results in a general framework. This framework allows the organisation of existing results and the identification of important gaps in our current understanding.

In what follows, the general framework will be introduced and then the research results within the framework will be elaborated from four aspects in Sections II-V. Finally, more interesting and promising research directions are envisioned in Section VI.

S. Knorn is with the Faculty of Engineering Sciences, Uppsala University, Uppsala, Sweden `steffi.knorn@signal.uu.se`. Z. Chen and R. Middleton are with the School of Electrical Engineering and Computer Science, The University of Newcastle, Callaghan, NSW 2308, Australia `zhiyong.chen, richard.middleton@newcastle.edu.au`. The work was supported by the Australian Research Council under grant No. DP130103039.

Usually, the dynamic behavior of an *individual* system is described by its trajectory in the time dimension denoted by a vector function $x(t)$ of time t. When a system of *multiple* agents is considered, however, a network dimension is added to its dynamic behavior. Specifically, the system behavior is denoted by a vector function $x_1(t,k)$ of both time t and agent index k. Here, $t \in [0, \infty)$ is the time variable in a continuous time setup or $t = 0, 1, 2, \cdots$ represents sequential time instants in a discrete time setup. The group of agents is labeled by $k = 1, 2, 3, \cdots$. Additionally, let $x_2(t,k)$ be the network influence, i.e., the external input originated from network, for the agent k. Now, the dynamic network behavior can be described in terms of $x_1(t,k)$ and $x_2(t,k)$ in two dimensions, i.e., the time dimension and the network dimension. In this sense, the model is termed as a two-dimensional (2D) model.

On the one hand, the system dynamics in the time dimension are represented by the following equation

$$\delta x_1(t,k) = f(x_1(t,k), x_2(t,k), t, k) \tag{1}$$

where the operator δ is defined as follows

$$\delta x_1(t,k) := \begin{cases} dx_1(t,k)/dt, & \text{continuous time} \\ x_1(t+1,k) - x_1(t,k), & \text{discrete time} \end{cases}.$$

On the other hand, the influence in the network dimension is as follows

$$\Delta x_2(t,k) = g(x_1(\tau(t,k), n(t,k)), t, k) - x_2(t, k-1) \tag{2}$$

where

$$\Delta x_2(t,k) = x_2(t,k) - x_2(t,k-1).$$

The vector $n(t,k)$ in (2) denotes the neighbors of k at time t; and the vector $\tau(t,k)$ denotes the (element-wise) corresponding time at which the state of the agent $n(t,k)$ is measured and contributes as network influence. In most scenarios, the network influence $x_2(t,k)$ is statically determined by the state of neighbor agents. In other words, the influence does not propagate in network dimension. This property is characterized by the term $-x_2(t,k-1)$ in (2) such that it reduces to

$$x_2(t,k) = g(x_1(\tau(t,k), n(t,k)), t, k). \tag{3}$$

For two vectors $a = [a_1, \cdots, a_r]^\intercal$ and $b = [b_1, \cdots, b_r]^\intercal$, we denote $x_1(a,b) := [x_1^\intercal(a_1,b_1), \cdots, x_1^\intercal(a_r,b_r)]^\intercal$.

Clearly, in a control design scenario, the system dynamics in (1) consist of plant dynamics and a designed controller. In particular, the state $x_1(t,k)$ is decomposed by $x_1(t,k) = [\xi^\intercal(t,k), \zeta^\intercal(t,k)]^\intercal$ where $\xi(t,k)$ is the plant state and $\zeta(t,k)$ the controller compensator state. With the corresponding decomposition of f, (1) is equivalent to the plant model

$$\delta\xi(t,k) = f_o(\xi(t,k), x_2(t,k), t, k, u(t,k)) \tag{4}$$

and the controller

$$\begin{aligned} u(t,k) &= \gamma(x_1(t,k), x_2(t,k), t, k) \\ \delta\zeta(t,k) &= \eta(x_1(t,k), x_2(t,k), t, k). \end{aligned} \tag{5}$$

Note that the controller (5) explicitly depends on the network influence $x_2(t,k)$ for achieving a desired collective behavior. However, the function f_o in (4) usually does not explicitly depend on $x_2(t,k)$ for an autonomous agent unless passive network coupling exists among the agents. The control design problem is to find the functions γ and η in (5), under prescribed constrains, such that the system state $x_1(t,k)$ satisfies desired requirements in both time and network dimensions.

Within the aforementioned framework, we will review the research outcomes in the literature from four perspectives: (i) agent dynamics represented by the function f_o in the model (4); (ii) network topologies, i.e., the neighborhood function $n(t,k)$ in (2); (iii) feedback and communication mechanisms that describe how neighbors influence an agent's dynamics through the measurement function g and the measurement time vector $\tau(t,k)$ in (2); and (iv) desired collective behaviors.

II. Agent Dynamics

In this section, the review focuses on the agent dynamics represented by the function f_o in the model (4). From this aspect, the research works can be categorized according to linearity and homogeneity of agents.

A. Linear Homogeneous Network

1) Fundamentals: In the simplest, homogeneous case, a group of N identical agents is considered in a continuous-time setup with (4) having a special single-integrator form

$$\frac{d\xi(t,k)}{dt} = u(t,k),\ k = 1, \cdots, N. \tag{6}$$

The controller (5) and the network influence (2) typically take the form

$$u(t,k) = x_2(t,k) = \sum_{j=1}^{N} a_{kj}(t)\left(\xi(t,j) - \xi(t,k)\right) \tag{7}$$

where $a_{kj}(t)$ is the (k,j)-th entry of the corresponding adjacency matrix $\mathcal{A}(t)$ which describes the connection graph among the agents. A Laplacian matrix $\mathcal{L}(t)$ can be defined based on $\mathcal{A}(t)$ such that

$$\frac{d\xi(t)}{dt} = -\mathcal{L}(t)\xi(t),\ \xi(t) = [\xi(t,1), \cdots, \xi(t,N)]. \tag{8}$$

A brief introduction on graphs, adjacency matrix, and Laplacian matrix can be found for instance in [13], [14]. The motivation to use (7) is that each agent converges towards the weighted average of the states of its neighbors. Such a collective behavior, called consensus, is described in the time dimension, i.e., along the direction $t \to \infty$ only. It is not concerned with the performance evolution in the network dimension, in particular, as the length of the network dimension is finite, denoted by N. A number of results established consensus of single-integrator systems and also more general double-integrator systems if the underlying directed graph satisfies an appropriate connectivity condition, see for instance [15], [16], [17], [18], [19], [20]. The most general form of this condition involves a directed spanning tree of a union of the time varying graph topologies, see for instance [13], [19], [21].

A number of extensions to this basic problem with single or double integrator dynamics have been studied for the general linear homogeneous systems

$$\frac{d\xi(t,k)}{dt} = A\xi(t,k) + Bu(t,k),\ y(t,k) = C\xi(t,k), \tag{9}$$

where A, B and C are constant matrices of appropriate dimension. A range of references (see for example [22], [23], [24], [25]) focused on finding feedback control laws such that consensus (in terms of the outputs $y(t,k)$) can be achieved. Discrete-time systems were studied in, e.g, [26] and [27].

In the standard consensus problem the network communication graph is described by an adjacency matrix $\mathcal{A}$ with nonnegative weights. The authors of [28] derived necessary and sufficient conditions for a network including antagonistic interactions (modelled as negative weights on the communication graph) to reach bipartite consensus, in which all agents converge to a value which is the same for all in modulus but not in sign.

2) Controllability and observability: To study reachability and controllability of a network, assume that a subset of nodes $\mathbb{I}_c \subset \{1, \cdots, N\}$ can be controlled by an external input $\mu(t)$, in addition to the local control input in (7). For a network of single integrators as in (8), adding $\mu(t)$ leads to

$$\frac{d\xi(t)}{dt} = -\mathcal{L}\xi(t) + \mathcal{B}\mu(t) \tag{10}$$

where $\mathcal{B}$ describes the influence of the external input onto the vector of agent sates $\xi(t)$. To enable the study of observability, assume that an external processor collects information from a subset of nodes $\mathbb{I}_o \subset \{1, \cdots, N\}$, that is,

$$\eta(t) = \mathcal{C}\xi(t) \tag{11}$$

where the output matrix $\mathcal{C}$ relates the information gathered from the processor to the vector of agent states. The processor aims to reconstruct the state of the entire network from $\eta(t)$.

Reachability and controllability of a network with first order dynamics and a single control node case was first introduced in [29]. This work gives a necessary and sufficient condition for network controllability by selecting a specific node as leader. Necessary conditions for controllability based on algebraic graph tools and the notion of equitable partitions of a graph were published in [30], [31], [9]. In [32] the same methodologies were used to study observability of general graphs. The authors proposed an estimation method based on the consensus algorithm and linear control theory and gave necessary observability conditions. While these results provide necessary conditions for general graphs, [33] and [34] provide necessary and sufficient conditions to characterize all and only the nodes from which the network system is controllable (reachable) or observable. Controllability of networks with a switching communication topology, with $\mathcal{L}$ in (10) replaced by $\mathcal{L}(t)$ explicitly depending on t, was studied in [35] showing that a controllable switched network can be made up of uncontrollable subsystems. Necessary and sufficient conditions to ensure controllability of systems with time delays and switching topologies were derived in [36].

3) Dynamic average consensus: In the model (10), $\mu(t)$ can also be regarded as time-varying reference signals to the agents. In this case, the problem that the agents aim to track the average of individually measured reference signals is referred to as dynamic average consensus in opposition to the static consensus discussed before. In other words, a snapshot of the input vector $\mu(t)$ is used to initialize the agent states in static consensus, after which the input is ignored, resulting the dynamics (8).

The early work was found in [37] using frequency domain analysis to guarantee zero steady-state error for ramp references. Other continuous-time algorithms can be found in [38], [39], [40] with robustness with respect to initialization errors. In [41], a discontinuous control algorithm was proposed for tracking bounded signals with bounded derivatives. Discrete-time approaches were studied in [42], [43] where bounds on the step size are computed for a desired steady-state consensus error. The further result in [44] studies robustness to initialization errors.

4) Robustness: Robust stability of multi-agent systems has been studied in [45] considering three different types of multiplicative perturbations. The results imply for instance that one of the worst case perturbations is an identical diagonal perturbation. Robust consensusability of networks of coupled single integrators subject to communication noise has been studied in [46]. It is shown that the problem is closely related to H_2 norm measures: systems with lower H_2 norms will remain closer to the consensus despite the presence of noise. A robust synchronization protocol to synchronize a network of controlled discrete-time double integrators with unknown model parameters and subject to additive measurement and process noise was proposed in [47]. Robust synchronization of networks of general linear input-output systems subject to additive perturbations of the transfer matrices is studied in [48]. A dynamic protocol that synchronizes the network for all additive perturbations within a given tolerance is designed.

Note that the robustness with respect to sampled or quantized measurement and input or feedback delays is further studied in Sections IV-B and IV-C. Some results studying robustness of heterogeneous or nonlinear networks are also discussed in Sections II-B and II-C.

B. Linear Heterogeneous Network

When studying heterogeneous dynamics, it is assumed that the dynamics of agent k are described by the local state space representation A_k, B_k, and C_k (instead of a general A, B, and C in (9)). This class of problem is usually referred to as a "synchronization problem". In the consensus literature, the system models are usually simple integrators and the focus lies on the communication graph, the traditional synchronization literature focusses on the individual dynamics (see the classification in, e.g., [22]). Recently, researchers aim to jointly study both individual dynamics (e.g. heterogenous and nonlinear agents) and communication graphs. Two typical research approaches are listed below.

1) Cooperative output regulation problem: The problem is formulated by the heterogeneous version of (9) subject to the exogenous signal $v(t)$, i.e.,

$$\begin{aligned} \frac{d\xi(t,k)}{dt} &= A_k\xi(t,k) + B_k u(t,k) + E_k v(t), \\ y(t,k) &= C_k\xi(t,k) + F_k v(t). \end{aligned} \qquad (12)$$

Specifically, $v(t)$ represents the reference input to be tracked or the disturbance to be rejected and it is generated by an exosystem $\dot{v}(t) = Sv(t)$. The objective is to design a distributed controller to guarantee asymptotic stability for $v = 0$ and the local tracking error $y(t,k)$ approaches zero for all k. The problem is different from a general synchronization/consensus problem in the sense that the final trajectory for each agent is specified by a real exosystem. A subset of the N agents are permitted access to the exogenous signal v for feedback control. The exosystem acts as the leader which all subsystems of the plant aim to follow. The cooperative output regulation problem of linear multi-agent system has been studied using feedforward control in [49], [50]. However, this method is unsuitable when investigating plant uncertainties for the agent dynamics. Therefore, the robust version has been studied in [51], [52] using an internal model approach.

2) Virtual exosystem approach: The general synchronization/consensus problem of heterogeneous version of (9), i.e.,

$$\frac{d\xi(t,k)}{dt} = A_k\xi(t,k) + B_k u(t,k),\ y(t,k) = C_k\xi(t,k) \qquad (13)$$

has been also studied by a virtual exosystem approach. The controller takes a special form of (5), i.e.,

$$\begin{aligned} u(t,k) &= H_k\zeta(t,k) + K_k x_2(t,k) \\ \frac{d\zeta(t,k)}{dt} &= F_k\zeta(t,k) + G_k x_2(t,k) \end{aligned} \qquad (14)$$

where $x_2(t,k)$ is in a special form of (3), i.e.,

$$x_2(t,k) = P_k \sum_{j=1}^{N} a_{kj}(y(t,j) - y(t,k)). \qquad (15)$$

It was shown in [53], [54] that it is necessary and sufficient for synchronization that all individual systems are able to track the same virtual exosystem $\dot{v}(t) = Sv(t)$, that is, the model of each individual system together with its local controller must embed an internal model of the virtual exosystem. Since the exosystem only exists as part of the individual system and its controller it is therefore referred to as a virtual exosystem. An alternative development was studied in [55] when system uncertainties are also taken into consideration. A similar idea was discussed in [56] that in order to synchronize heterogeneous agents on some trajectory, all agents together with their local controller must include the model of the reference trajectory.

C. Nonlinear Dynamics

The early research on synchronization was mainly for simply coupled nonlinear systems. The complexity is from nonlinear characteristics. Typical examples include synchronization of chaotic coupled networks; see [57], [58], [59], among a huge number of others. Relevant works were also found in research of nonlinear oscillator theories including the Malkin theorem for phase coupled oscillators [60], multivariable harmonic balance [61], and the contraction analysis for global convergence[62].

The complexity of these works is not from network structures. When more attention is paid on complex networks, there have arisen a large body of works, e.g., [63], [64], focusing on synchronization of complex networks (such as scale-free or small world networks) where network nodes are described by nonlinear dynamics.

The synchronization/consensus problem studied for nonlinear multi-agent systems is different from that for conventional "complex networks" although there is no strict division. Specifically, the former is more concerned with control design methodologies. For instance, it aims to seek a decentralized controller $u(t,k)$ for a multi-agent system of time-varying nonlinear dynamics

$$\frac{d\xi(t,k)}{dt} = \phi(\xi(t,k),t) + Bu(t,k), \tag{16}$$

with a nonlinear function ϕ, such that collaborative behavior is achieved under a certain network topology.

A number of results depend on some restrictions on nonlinearities. For example, the consensus problem has been studied under a class of globally Lipschitz condition

$$\|\phi(\xi_1,t) - \phi(\xi_2,t)\| \le \rho\|\xi_1 - \xi_2\|,\ \forall \xi_1, \xi_2. \tag{17}$$

It was shown in [65] that a simple linear controller relying only on information from direct neighbors still guarantees stability for first order systems. The result also holds for second order systems [66], [67] and more general systems [68], [69]. Within the same setup, the globally Lipschitz condition can be removed when a consensus problem is studied with semi-global stability [70]. Nonlinear protocols were also studied in [71] for the dynamic average consensus problem discussed in Section II-A3 when the nonlinearities satisfy a modified Lipschitz condition.

For a network of agents with nonlinear heterogeneous dynamics of the form

$$\begin{aligned} \frac{d\xi(t,k)}{dt} &= \phi_k(\xi(t,k),t) + \varphi_k(\xi(t,k),t)u(t,k), \\ y(t,k) &= h_k(\xi(t,k)), \end{aligned} \tag{18}$$

velocity consensus controllers based on an input-output passive assumption were proposed in [72]. The design of simple but robust output feedback controllers to achieve position consensus was studied in [73]. The paper deals with nonlinear heterogeneous agents with relative degree two and stable zero-dynamics, that satisfy local and global passivity assumptions. Suitable controllers were proposed in [74] to guarantee that a network with agents that are input-output passive with a radially unbounded positive definite storage function and a strongly connected communication graph reaches output consensus.

There are other approaches for dealing with multi-agents with nonlinear heterogeneous dynamics. Nonlinear controllers were designed for second order systems in [75], [76] using adaptive control and small gain theorem, respectively, without imposing the aforementioned global Lipschitz or passivity assumption. The results presented in [77] for a group of linear cooperative systems can be applied to nonlinear systems [78] if the cooperative control Lyapunov function designed for the linear systems also satisfies special differential inequalities. A Lyapunov technique is used in [79] studying the synchronization of distributed non-identical unknown nonlinear dynamics to prescribed nonlinear and unknown target dynamics.

The synchronization problem was also discussed for the special nonlinear heterogeneous systems described by Euler-Lagrange equations. For example, [80] developed suitable estimation and control strategies to track a dynamic leader. The controllers proposed in [81], [82] guarantee synchronization or flocking of Euler-Lagrange systems with uncertain kinematics and dynamics or uncertain parameters, respectively. Distributed adaptive control algorithms were proposed in [83] to ensure synchronization using velocity information only. More research on this topic dealing with delays will be further discussed in Section IV-C.

Recently, more systematic approaches have been proposed for synchronization of complicated nonlinear heterogeneous multi-agent systems, e.g., in lower triangular form. For example, the cooperative output regulation in Section II-B1 has been generalized to this scenario in, e.g., [84], for a leader-following network. However, a group of multiple agents is treated as a bulky multiple-input multiple-output system and the design approach is centralized.

The virtual exosystem approach in Section II-B2 also has its development for nonlinear systems. The simple extension beyond [53] can be found in [85] under some conditions applied to the nonlinearities. More complicated situations were reported in [86], [87], [88], [89]. A common procedure for these works is a two-step manner thanks to properly designed reference models: (1) regulation of each individual agent's output to its reference model, and (2) consensus of reference models. Specifically, each reference model must embed a virtual exosystem that describes the desired synchronization pattern. The resulting regulation problem was studied in [86] using feedforward design. The robust case was further investigated using the internal model principle. In [87], the internal model for each agent is based on the centralized network of all reference models. The beauty of the approach in [88], [89] is that the internal model for each agent can be done individually in a completely decentralized manner.

III. Network Topologies

Whether a distributed network reaches consensus using a decentralized control law strongly depends on the topology of the communication network among the agents. The research on network topologies is concerned with the characteristics of the neighboring function $n(t,k)$ in (2). For fixed topologies, the function $n(t,k)$ does not depend on t; for switching topologies, the function $n(t,k)$ varies with t but takes values only from a finite set; and for time-varying topologies, the function $n(t,k)$ is defined more generally, usually relying on the network status. Also, the special leader-follower topology and research on preservation of connectivity are discussed in this section.

A. Fixed Topologies

It was shown in [15] that, for an undirected network of double-integrators, velocity consensus can be achieved if the fixed graph is connected. Later, [18] systematically analyzed the problem that a network of single-integrators reaches average consensus if the graph is connected (undirected network) or strongly connected and balanced (directed network). The problem was also studied in [13] that with a fixed topology and constant weighting factors, a directed network of first order dynamics asymptotically achieves consensus if and only if the graph has a spanning tree. The result was extended to a directed network of second order dynamics in [19], [20].

Some networks with fixed topologies in specific forms were studied in the literature. The authors of [33], for example, investigated the reachability and observability properties of a network system focussing on cases where the communication graph is a path or a cycle. A necessary and sufficient condition was published in [90] for diagonal stability for systems with a "cactus" structure (i.e. a pair of distinct simple circuits have at most one common vertex). The authors of [91] investigated how to reconstruct a tree-like topological structure of a network of linear dynamic systems.

B. Switching Topologies

In a network of distributed agents, a certain number of edges may be added or removed from the graph under varying circumstances, which results in non-fixed topologies. In the area of non-fixed topologies, the term "switching topologies" describes the case where the topology changes over time but only switches between a finite, known set of distinct communication graphs. In contrast, "time-varying topologies" include all networks where an infinite set of arbitrary graph structures is considered.

The paper [18] also studied directed networks with switching topologies (apart from a fixed topology, see Section III-A). Consider a finite collection of strongly connected and balanced digraphs. A network of simple-integrators with switching topologies taken from the collection can asymptotically achieve average consensus for any switching signal. A weaker condition was proposed in [13] showing that consensus can be achieved asymptotically if the union of the collection of interaction graphs across some time intervals has a spanning tree frequently enough. The result was extended to a network of double-integrators with an undirected graph in [92].

C. Time-varying Topologies

One of the most interesting (and well studied) scenarios resulting in time-varying topologies is the so-called "nearest neighbor rule". Each agent interacts with all and only all agents within its limited sensing/communication radius. Such networks always have an undirected neighborhood graph. (In this sense, these results were weaker than those for directed networks with switching topologies discussed above.)

In [93], Vicsek et al. proposed a simple but compelling discrete-time model of autonomous agents using nearest neighbor rule. A theoretical analysis of discrete-time (linearized) Vicsek model can be found in [17]. It shows that consensus is achieved if there exists an infinite sequence of continuous, nonempty, and bounded time intervals such that the union of the collection of time-varying undirected graphs across each time interval is connected (called joint connectivity condition). Consensus of a network of continuous-time double-integrators with time-varying topologies due to a nearest neighbor rule was also studied in [16]. It was proven that as long as the graph remains connected at all times, the network achieves consensus regardless of switching in the neighborhood graph.

More complex flocking behavior in lattice-shape was studied for the same network in [94] also assuming the network is connected at all times. The problem was further studied in [95] considering a weaker joint connectivity condition. Some variations of the joint connectivity condition defined by the integral of adjacency matrix over a certain time interval were studied in [96], [97]. Other than the joint connectivity condition, it was also proved in [98] that convergence (not necessarily to consensus) can be guaranteed if the time-varying topologies are cut-balanced: if a group of agents influences the remaining ones, the former group is also influenced by the remaining ones by at least a proportional amount.

Time-varying topologies can also be caused by active network weight tuning. To guarantee stability in complex networks, adaptive strategies are proposed to appropriately tune the strengths of the interconnections among network nodes, see, e.g., [99], [100].

D. Leader-Follower Topology

In the networks discussed above described by directed graphs, the leader following scenario can be seen as a special case. In case there exists one agent in the group without any incoming links, this agent can be regarded as the leader of the network. The existence of a leader is essential in some scenarios such as distributed tracking control of multi-agent system where the reference trajectory is set by an active leader. The authors in [101], [102] designed a suitable neighbor-based local controller together with a neighbor-based state-estimator to track an active leader whose velocity is unknown to the agents. Other researchers proposed suitable distributed tracking control laws for networks of first-order agents [103] and general linear systems [104], [105]. The work in [106] studies the tracking problem of a dynamic virtual leader via a variable structure approach considering that only partial measurements of the states of the leader and the followers are available.

Several results are available for robust leader-following algorithm design subject to measurement noise (e.g., [107]), model uncertainties (e.g., [80], [108]) or delays (e.g., [109]). Note that in the cooperative output regulation problem studied in Section II-B1, the exosystem acts as the leader which all the agents aim to follow.

In leader-follower topologies, some authors developed consensus algorithms based on so-called "pinning control". This refers to the approach that only a small fraction of agents have access to the reference state. These can be regarded as leaders or they are pinned by other leaders. Some examples of pinning control for networks of single integrators can be found in [110], [111] and double integrators in [20]. One of the most interesting problems in the area of pinning control of complex networks is to choose the best set of pinned nodes (see, e..g, [112], [67]).

E. Preservation of Connectivity

It has been discussed that different connectivity assumptions are required to ensure consensus or synchronization of a network. A more practical problem is how to ensure these assumptions, or, how to preserve network connectivity. A simple idea is to define a potential field and a control algorithm forces the network to move in the direction of the negative gradient of the potential field so that network connectivity can be preserved. A centralized control approach (requiring global information of the underlying graph) based on a potential field was developed in [113]. The connectivity of the system is treated as an imaginary obstacle in the free space, and artificial potential fields are used to avoid collisions with it.

In [114], a distributed topology control protocol that decides on both deletion and creation of communication links between agents was designed. The distributed control law proposed in [115] to ensure edge maintenance allows the control forces to go towards infinity if the distance between two agents get close to a critical distance (above which the link would break). To avoid the infinite forces, bounded control laws were developed in [116], [117], [118], [106]. Preservation of connectivity was also studied as a part of network integrity (defined as the ability of the network to support a desired communication rate) in mobile robotic networks in [119].

Often, preservation of connectivity is studied together with collision avoidance as both problems can be dealt with using the same techniques. For example, potential functions can also be used for designing a "repulsive potential force" for collision avoidance. In [120], a control law was achieved by combining a repulsive potential (to ensure collision avoidance) and an attractive potential (to ensure convergence). A protocol was presented in [121], [122] to achieve flocking only requiring suitable conditions on the initial states of the flock while, at the same time, guaranteeing collision avoidance. The control law proposed in [95] achieves uncrowded flocks with collision avoidance during the entire evolution process through a repulsion mechanism.

However, for stationary obstacle avoidance, using repulsive potential fields is usually a challenging task. A navigation function using artificial potential fields was developed in [123], [124] to navigate a robot through a field with spherical obstacles. The navigation function framework was then extended to multi-agent systems for obstacle avoidance in results such as [125], [126]. The navigation function in [127] is also suitable for stationary obstacle avoidance while at the same time maintaining global network connectivity. Potential fields or navigation functions are also used in [128], [129] to achieve obstacle avoidance while the agents must achieve a cooperative network objective such as formation control or consensus.

Instead of applying the control laws discussed above to ensure connectivity preservation, externally applied boundary conditions (e.g. periodic boundary conditions and rebounding conditions) can also be used for the same purpose. It was shown in [130] that by applying periodic boundary conditions to a Vicsek model, connectivity can be achieved. More general theoretical and experimental analysis can be found in [131], [132]. It was shown that a group of agents in a bounded plane can be almost always jointly connected and therefore form a complete flock.

IV. Feedback and Communication

Feedback and communication mechanisms describe how neighbors influence an agent's dynamics through the measurement function g and/or the measurement time vector $\tau(t,k)$ in $x_2(t,k) = g(x_1(\tau(t,k), n(t,k)), t, k)$. Measurement output feedback control is concerned with the circumstances when an agent state x_1 is not completely measured or transmitted in the network. So, the function g selects the available measurement output. For sampled data control, the function $\tau(t,k)$ takes isolated sampling instances; and for quantized control, the function g is of a quantization form. A more general selection of $\tau(t,k)$ also accommodates communication delays.

A. Measurement Output Feedback Control

In many realistic multi-agent systems, usually not all states of all agents are measurable due to practical limitations or to save costs. In these cases measurement output feedback control becomes an important research topic. Usually an observer design strategy for the agents is required. In the early paper [133], a Nyquist criterion was used to study the observer design of a linear homogeneous consensus problem. One special scenario of measurement output feedback control covers networks of double integrator dynamics without velocity information. In [101], [102] a leader-follower system was studied when the leader velocity is unknown. A more general undirected communication topology was considered in [20] assuming that the double integrator agents cannot access relative velocity measurements.

Two different design methods for double integrator systems without velocity measurements and the additional requirement that control inputs must be a priori bounded were proposed in [134]. Connectivity preserving algorithms for networks with switching topologies based only on position measurement outputs were studied in [135]. The containment control problem (i.e. to drive the followers into the convex hull spanned by the dynamic leaders) was studied using only position measurements in [136]. The synchronization problem was also studied for spacecraft systems without velocity measurements in [137], [138].

Research interest in consensus problem has also been devoted to high-order agents. Some results such as [139], [140] require the knowledge of all the states of neighboring agents. In [141] the proposed control input has the same dimension as the corresponding state space. The output feedback consensus problem was considered in [22]. Even though the exact measurements of all states are not necessary, the proposed controller requires the knowledge of all state estimates of the neighbors. Hence, the quantity of the transmitted information needed is identical to the state feedback case.

Various conditions guaranteeing consensus of linear homogeneous systems by static output feedback can be found in [142]. How to construct a suitable dynamic output feedback controller for more general consensus problems was studied in a unified manner using observer-based compensator in [23] (for homogeneous systems) and [54], [55] (for heterogeneous systems).

B. Sampled and Quantized Control

In multi-agent systems sampled data are often only sent to the neighbors periodically at discrete time instances. A framework for studying consensus problem of multi-agent systems via sampled control was introduced in [143] and [144] for a fixed topology and switching topologies, respectively. Two sampled data based discrete-time coordination algorithms were studied in [145] and [146] which gave necessary and sufficient conditions on the interaction graph, the damping gain and the sampling period to guarantee coordination. Independent and asynchronous sampling times in a directed network of continuous-time second-order agents were considered in [147], [148]. Hybrid dynamic system theory was used in [149] to propose an allowable upper bound of the sampling period for a network with random switching topologies.

In fact, due to the bandwidth constraints in communication, the data must not only be sampled but also quantized before transmission. The authors of [150] proposed a quantized gossip algorithm, that forces the network to converge to a set of quantized consensus distributions for an arbitrary initial vector and arbitrary connected graph. The term gossip algorithm describes a control protocol where at each time instant exactly one agent updates its state based on the information transmitted from only one of its neighbors. The integer-valued gossip algorithm was also studied in [151]. An extension of the gossip algorithm covering cases where the quantization is uniform, and the initial values of the agents are reals (as opposed to being integers) was studied in, e.g., [152], [153].

The average-consensus problem with real-valued states and quantized communications was also considered in other literature. For instance, a critical quantizer accuracy (independent from the network dimension) was also derived in [154] to guarantee convergence, the finite-level quantization problem was discussed in [155], a quantized-observer based encoding-decoding scheme was designed in [156], and necessary and sufficient conditions on sampling period and design parameters were obtained in [157].

C. Communication Delays

In realistic system settings, the exchange of information on a network introduces delays between agents. These delays are often disregarded in the analysis. Hence, it is an important question whether known algorithms also achieve consensus in the presence of delays, i.e. if the consensus is robust to delays. The three most commonly studied delay models are constant delays, time-varying delays and distributed delays.

The robustness of consensus in discrete-time single-integrator multi-agent systems to arbitrarily large delays (bounded by some arbitrary bound) was discussed for instance in [158] showing that consensus is reached exponentially fast if the graphs are repeatedly jointly rooted. In a similar problem setting [159] proved that if there exists an arbitrary upper bound for which in this time the union of graphs has a spanning tree, then consensus is achieved.

For continuous-time networks with continuous-time communication, delay robustness has been investigated in the frequency domain using small gain arguments or the generalized Nyquist criterion. For instance, in [160] consensus is ensured if the closed-loop transfer function of each agent has unit gain at DC and gain strictly less than 1 elsewhere, the information graph has a globally reachable node, and the information delays are finite constants. In [161] necessary and sufficient conditions were provided ensuring local or global exponential convergence for finite delays and a closed form expression was derived for the final consensus. In [162] scalable, simple, and accurate set-valued conditions were provided for consensus and conditions were derived for the convergence rate of a single integrator network with feedback delays. The results are also suitable for nonlinear coupling functions and switching topologies in [163].

In the time-domain, small-μ analysis has been used to study delay robustness in [164] while [165] based their analysis on integral quadratic constraints and [166] modelled a multi-agent network as PDEs to mimic PDEs. However, these results consider exclusively networks with fixed topologies.

Heterogeneous agents with heterogeneous delays were dealt with in existing results. It was shown in [167] that for identical integrators with heterogeneous delays, consensus is robust against any finite constant delays. The work in [168] presented conditions for consensus of heterogeneous linear systems with heterogeneous delays which are robust and scalable to unknown, arbitrary large topologies and unknown, bounded delays. Networks of Euler-Lagrange systems with heterogeneous delays were investigated in [169], [170], [171], etc.

Many researchers working in the field of consensus of multi-agent systems with delays use sums of Lyapunov-Krasovskii functionals or Lyapunov-Razumikhin functions. The former has been applied to investigate single integrator networks in [172], [173], multi-agent networks of passive agents or nonlinear agents with relative degree one in [174], [175], and multi-agent systems with relative degree two in [73]. The main disadvantage of using Lyapunov-Krasovskii functionals is that the underlying graph needs to be undirected or weight-balanced. Lyapunov-Razumikhin functions are used to obtain results for more general multi-agent systems of single integrators and directed, uniformly quasi-strongly connected graphs in, e.g., [176].

V. Collective Behaviors

The discussion in the previous three sections focused on variations of consensus or synchronization behaviors of multi-agent systems. Such behaviors are described in the time dimension, i.e., along the direction $t \to \infty$ only (in particular, the size of the network dimension is finite, N). In the time dimension, more complicated collective behaviors, e.g., in formation, are discussed in this section. Moreover, there exist some other research works which are concerned with collective behaviors in both the time dimension and the network dimension assuming the network size may approach infinity.

A. Formation Control

In formation control one seeks to design a suitable controller to ensure a group of agents move through space in an ordered manner along a desired reference trajectory or path while sometimes avoiding collisions with obstacles or other agents. For instance, seeking to navigate a group of robots to waypoints, while avoiding hazards and keeping formation, three different formation forms (line, diamond, wedge) and two reference techniques (leader reference, unit center) were compared in [177]. Three approaches in formation control (leader following, behavioral approach and virtual structure) were combined in [178] to control a formation of spacecrafts.

In [179] it was shown that formation stabilization to a point is feasible if and only if the sensor digraph is globally reachable. Similar results were also obtained for formation stabilization to a line and to more general geometric arrangements. Geometric conditions for feasibility of formations of non-holonomic vehicles were presented in [180]. Later, a decentralized gradient control law to stabilize a group of point mass robots to any formation corresponding to an infinitesimally rigid framework was proposed in [181].

In [182] and [183] formations of unicycles in cyclic pursuit were considered. The objective is to find control laws such that a group of agents rotates around a common beacon with the same angular velocity and identical distances between neighboring agents. Further discussions on problems of synchronization for systems of particles modelled as kinematic unicycles can be found in [184]. The results relay on an all-to-all communication assumption. In [185], the same authors generalize their design of artificial potentials such that the resulting control laws respect the communication constraints. A similar formation control problem was also studied in [186], where the formation has a precise geometric description in terms of the desired ordering and the spacing between the vehicles. Instead of assuming all-to-all communication the authors in [187] developed a control algorithm to guarantee the global asymptotical stability of the circular motion around a virtual reference beacon with prescribed direction of rotation. Later, no-beacon collective circular motion of jointly connected multi-agents was studied in [188] and [189]. In particular, an algorithm was proposed not relying on global information including a reference beacon, a common reference frame, agent labels, or agent homogeneity. The approach has been further extended to more complicated motion patterns in [86], [190].

The multi-agent rendezvous problem is another important research topic in formation control which requires all agents to meet at one point. An early formulation and algorithmic solution of the rendezvous problem was introduced in [191] for the robots of a limited sensor range. The results were extended to stop-and-go strategies in [192] and [193]. In [194] proximity graphs were used to solve an n-dimensional rendezvous problem in the presence of link failures. A simple quantized control law was designed in [195] for Dubins car agents of limited sensors which reports only the presence of another agent within some sector of its windshield. It was shown that agents achieve rendezvous given a connected initial assignment graph without relying on any estimation procedure to reconstruct coordinate information.

Another fundamental task in the area of formation control is formation shape control. The aim is to design decentralized control laws for each agent to restore a formation shape in the presence of small perturbations from the desired shape. In case agents can actively control the distances towards their neighbors the graph rigidity of the information graph becomes the crucial concept for the formation shape maintenance problem. For instance, [196] discussed how to add or remove vertices to ensure that the graph stays rigid and was further extended in [197] which developed methods to construct rigid point formations. An algebra was introduced in [198] that formalizes performing some basic operations on graphs and allows creation of larger rigid-by-construction graphs by combining smaller rigid subgraphs. The work in [199] demonstrated how control based on distance preservation can be achieved in the presence of a cycle. An analysis of the characteristics of global convergence properties of a directed rigid formation (acyclic directed formations) can be found in [200]. A decentralized control law was introduced in [201] that maintains the formation shape by controlling certain inter-agent distances, where only one agent is responsible for maintaining each distance.

B. Scalability of Networks

Scalability of networks is concerned with collective behaviors in both the time dimension and the network dimension (i.e., 2D) assuming that the network size may approach infinity. The existing research works are relatively rare compared to the mature investigation of collective behaviors in time dimension only. A typical formulation in this research area is string stability of vehicle platoon. The terms vehicle platoon or vehicle string usually refer to a group of N vehicles (e.g. platoon or string) that is required to follow a given reference trajectory while the vehicles keep a prescribed distance to neighboring vehicles.

In most cases, it is straightforward to design decentralized controller to achieve stability of a string in the time dimension. Thus, small initial deviations or disturbances cause small perturbations. However, it is well known that error signals can amplify when travelling through the string, in network dimension, resulting in growth of the local error norm with the position in the string. This effect is referred to as 'string instability' [202], [203], or 'slinky effect' [204], [205].

In unidirectional strings (information propagates through the string in one direction only), [204] showed that using a distance that depends on the velocity (e.g., time headway policy) instead of a fixed distance between the vehicles can ensure string stability if the time headway is chosen sufficiently large. Another strategy was presented in [206] and [207], where the authors design a local controller that depends on the position. When the velocity or the acceleration of the lead vehicle or the reference velocity is propagated to each vehicle within the platoon, string stability can also be guaranteed, see e.g. [203] or [208].

The authors of [203] examined a bidirectional string (wherein the neighborhood function includes the preceding and following agents) with constant spacing and showed that string stability can be achieved with sufficiently large coupling with the leader position. The authors of [208] approximate a linear, bidirectional string of N vehicles as a PDE. It is shown that the least stable eigenvalue of the PDE approaches the origin with $O(1/N^2)$ if the string couplings are symmetric and $O(1/N)$ if the couplings are asymmetric. These results were later extended in [209] where the least stable eigenvalue of the overall system matrix is studied. A similar approach in [210] using local controller revealed that the best performance is achieved with the optimal localized controller that is both non-symmetric and spatially-varying. Sufficient conditions for string instability of bidirectional, heterogeneous strings were derived in [211]. The authors of [212] proposed distributed receding horizon control algorithms for platoons of vehicles with nonlinear dynamics.

A range of different methods has been used in the literature so far to analyze vehicle platoons. The Laplace transform with respect to time was used in [213], [214], [211] to analyze the system dynamics in the frequency domain. Lyapunov Theory was applied in [205] and graph theory was used in [215] to analyze a string of vehicles with a general interconnection or communication structure. In [208] a bidirectional string was approximated as a PDE. Some classes of bidirectional vehicle strings can also be conveniently modelled using port-Hamiltonian system theory as shown in [216], [217]. A unidirectional vehicle string was systematically formulated as a 2D system in [218]. Some researchers also investigated the influence of time delays in vehicle platoons (e.g., [219]) and the effects of communication losses (e.g., [220]). But how these control laws perform with increasing string sizes (string stability) was not discussed.

Scalable robust stability analysis is also an interesting topic that aims to find conditions guaranteeing system-wide robust stability. In case these conditions are satisfied it can be guaranteed that the resulting analysis and design tools can easily be scaled as the network size increases. This is also referred to as "scalable robust stability criterion". Results for linear heterogeneous systems were presented in [203], [221], [222]. For instance, the criterion proposed in [221] shows that if all subsystems are SISO systems, the stability criterion has a graphical interpretation which resembles the classical Nyquist criterion.

VI. Future Works

The research on collective control of multi-agent systems is a broad area. From the four aspects discussed in this paper, there are many open research topics. Here, we would like to propose three major research directions.

First, there still exist many challenges in investigating the synchronization problem of multi-agent systems of nonlinear and heterogeneous dynamics in a complicated network. For example, different UAVs may exist in a flight formation control, and substations in a smart grid may have very different load and supply characteristics. Developing distributed control methodologies for nonlinear and heterogeneous multi-agent systems is of paramount importance for real-world applications. In fact, for multi-agent systems of homogenous dynamics, when synchronized, all agents have their behaviors automatically governed by the "homogenous kernel". But for multi-agent systems of heterogenous dynamics, the situation changes as such a "homogenous kernel" does not exist. In all existing results (see those reported in Sections II-B and II-C), it is required that all agents synchronize either to the trajectory specified by a leader or to a specified motion pattern determined by a virtual exosystem in a leaderless scenario. It is interesting but challenging to see how heterogenous agents are able to achieve a synchronization pattern that is not prescribed but automatically matching to agent dynamics and adaptive to environmental variations. It is not difficult to image that the problem becomes more complicated if nonlinear dynamics are concerned. An effective approach has yet to be developed.

Secondly, most existing works assume that every agent is autonomous. The coupling among agents is only introduced with the designed cooperative control. However, in many real scenarios, direct physical coupling exists among agents. It is represented by the dependence of f_o on $x_2(t, k)$ in (4). In a power network, each bus is coupled to neighboring buses through the so-called tie lines. Therefore, the group behavior is influenced by both the physical coupling and the designed cooperative connection. In the literature, the research on physical coupling of multi-agents is rare. The research on this topic is closely related to that on large-scale systems, but the research may have more focuses on network behaviors.

Thirdly, the most important feature of multi-agent systems is their autonomous nature (unless coupling exists as formulated in the previous case), which demands distributed operations. It is this feature that allows a multi-agent system to be scalable in the sense that, when the network size increases, similar global control objectives can still be achieved without increasing the complexities for sensing, communication, computing and control. Therefore, it is a promising topic to further study the behavior of a multi-agent system in both the time dimension and the network dimension. As reviewed in this paper, 2D collective behaviors have been studied in very rare scenarios in Section V-B. It deserves thorough investigation in the future for more general perspectives.

Finally, we would like to acknowledge that the research on multi-agent systems has experienced rapid development over the past two decades. Only a small proportion of literature was discussed in this paper. Among those publications not mentioned in this paper, some results are of importance to the development of the area. In particular, this paper only focuses on collective control methodologies and it does not touch some closely relevant areas including decentralized optimization and cooperative localization. Also, it is worth mentioning that the research discussed in this paper is intersected with other research disciplines such as neuronal network in biology, big data in computer science, social network in sociology, and so on.

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