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Predictive Transmission Control in Multi Sensor Estimation over Wireless Channels using Energy Harvesting and Sharing

Steffi Knorn[1*], Subhrakanti Dey[1], Anders Ahlén[1], and Daniel E. Quevedo[2]

[1]*Department of Engineering Sciences, Uppsala University, Sweden*

[2]*Paderborn University, Germany*

SUMMARY

We investigate the power control problem for multi sensor estimation of correlated random Gaussian sources. A group of wireless sensors obtains a local measurement and transmits their measurements to a remote fusion centre (FC), which reconstructs the measurements using the minimum mean-square error (MMSE) estimator. All sensors are equipped with an energy harvesting module and a transceiver unit for wireless, directed energy sharing between neighboring sensor. The sensor batteries are of finite storage capacity and may be subject to energy leakage as well. Our aim is to find optimal power control strategies, which determine the energies used to transmit data to the FC and shared between sensors, and that minimize the long term average distortion over an infinite horizon. We assume centralized causal information of the harvested energies and channel gains, which are generated by independent finite-state stationary Markov chains. The optimal power control policy is derived using a stochastic predictive control formulation. We also investigate a Q-learning based sub-optimal power control scheme and two computationally simple heuristic policies.

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KEY WORDS: multi sensor estimation, energy harvesting, energy sharing, power control, fading channels, Q-learning, networks

*Correspondence to: steffi.knorn@angstrom.uu.se

1. INTRODUCTION

Wireless sensors have become much more powerful and affordable in recent years. Hence, they are used in a growing number of areas such as environmental data gathering [1], industrial process monitoring [2], mobile robots and autonomous vehicles [3], and for monitoring of smart electricity grids [4]. Often, several sensors are used to construct a wireless sensor network. Each sensor wirelessly transmits its measurements over a network to a remote fusion center (FC), which further processes the received data, e.g., by reconstructing the measured sources or computing an actuation signal.

When using battery powered sensors, a significant challenge is to spend the available power in an optimal fashion. This is usually refereed to as "power control" in the control literature, see [5–11]. Some works also investigated the benefits of power control in combination with coding schemes as in [12].

Another promising alternative might be to harvest energy, which has attracted mostly researchers in the are of wireless communications. Energy can be harvested from the sensors' environment using, e.g., solar panels, windmills, thermoelectric elements, radio frequency energy harvesting units or vibration harvesters. However, since harvesting is an often unpredictable and unreliable power source and the rechargeable batteries have limited capacity, spending the available energy in an optimal fashion is a challenging task. Several optimal power control policies for different system settings with energy harvesting and optimizing a variety of performance criteria have been proposed.

For instance, power control policies to maximize the throughput or minimize the mean delay for a single transmitting device were presented in [13]. The work in [14] derived power control algorithms that maximize the mutual information of a wireless link considering either causal or non-causal side information. An optimal packet scheduling problem for a single-user communication system with infinite battery and energy harvesting capabilities considering that data packets and energy packets arrive at the transmitter in a random manner was investigated in [15]. More precisely, [15] develop optimal off-line scheduling policies for minimizing the delivery time for all packets to

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the destination in a deterministic setting with non-causal information. Finite horizon throughput maximization and the related problem of minimizing the transmission completion time for a given amount of data were studied in [16]. The effect of energy harvesting in optimal power control for source acquisition/compression and transmission was studied in [17, 18]. The work reported in [19] studied how to jointly control the data queue and battery buffer to maximize the long-term average sensing rate of a wireless sensor network with energy harvesting. The problem of designing optimal sensor transmission power control schemes under energy harvesting constraints has also been investigated in [20].

The authors of [21] considered an energy harvesting sensor, that sends it measurements towards a remote estimator, and developed a communication scheduling strategy for the sensor and an estimation strategy for the estimator that jointly minimize the expected sum of communication and distortion costs over a finite time horizon. A setting where sensor measurements are wirelessly sent from an energy harvesting sensor to a remote estimator has also been investigated in [22], where the authors developed an optimal energy allocation strategy such that under energy constraints due to the harvesting sensor, the distortion at the receiver is minimised. The recent work in [23] extended those results to a closed control loop, where the state estimates from the Kalman filter at the smart wireless sensor are sent via a wireless packet dropping link to the controller.

Apart from energy harvesting, wireless energy transfer is another promising option to overcome the limitations of finite power resources. Since wireless energy transfer is becoming more efficient and less costly, it can potentially be used to recharge batteries of wireless sensors. The authors of [24] showed through experiments that energy can be efficiently transferred between two resonant objects with efficiencies of over 50% for distances up to 2 meters. By choosing different resonant frequencies between each pair, which are coupled by an energy transfer link, it is hence possible to allow for highly efficient energy transfer. Similar energy transfer techniques were also discussed in [25]. Optimal control of energy and information transmission through wireless communication channels has also been of interest. Important works along this direction include [26–30], where

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the energy is assumed to be broadcasted in all directions, in contrast to the techniques discussed in [24, 25].

The important question of what benefits wireless energy transfer could bring to wireless sensor systems has already attracted some attention. In particular, [31] and [32] considered a wireless sensor network with a fixed base station and a wireless charging vehicle driving from sensor to sensor assuming wireless energy transfer as in [24]. An optimal power control scheme was derived and multiple necessary conditions for optimality are given for throughput maximization for a two-hop relay channel with one-way energy transfer from the source to the relay in [33].

A significant hurdle when using batteries, or other energy storage options such as capacitors, to power wireless sensors is the fact that these devices are not perfect. To address such issues, capacitor leakage aware algorithms for energy harvesting wireless devices were developed and successfully evaluated in [34, 35]. The approach in [36] considered a single communication link with a hybrid power source including a constant energy supply and energy harvesting prone to energy leakage. The authors in [37] studied throughput maximization of a single communication link, where the transmitter has full noncausal information of the fading channel gains and harvested energy but harvested energy is randomly lost. A slightly different approach was studied in [38], where saving harvested energy in the battery is assumed to be prone to losses whereas storing and retrieving the energy from the battery is considered lossless. In this situation, the optimal power control scheme is found to be a double-threshold policy.

Following a different line of research in [39] we investigated a multi sensor estimation problem via a star network of wireless sensors that report their measurements over temporally independent block fading channels to a central FC, which reconstructs the random source observed by the sensors. All sensors are equipped with individual energy harvesting modules and can in addition transfer energy via directed wireless links to neighboring sensors. Optimal power control policies for information transmission and energy sharing were derived to minimize the overall distortion at the FC over a finite time horizon. Considering a finite time horizon in [39] allowed to derive several necessary optimality conditions and led to a suitable heuristic power control scheme. However,

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implementing such finite time solutions in practice is usually undesirable. It is often not known how long a system has to function or the time horizon is very long, which leads to prohibitively big computations. The results in [39] are further unrealistic as they consider perfect batteries and/or capacitors. Moreover, for simplicity, the channel gains and harvested energies were assumed to be independent and identically distributed (i.i.d.) in [39] and only a single source was considered.

The present manuscript extends the results reported in [39] along the following lines:

- We study the more practical relevant and realistic case of infinite time horizon power control, which leads to a stationary power control scheme. For any application, the solution is independent of the time horizon, can be implemented based on causal information only, and does not require recalculations as long as the statistics of the underlying random processes remain unchanged.
- The sensors in this paper are assumed to measure a field of correlated sources (instead of the same point source).[2]
- The fading channels and harvested energies are described by finite state Markov chains (instead of i.i.d. channel gains and harvested energies).
- The sensor batteries/energy storage devices are prone to energy leakage.

The current work contributes in the following ways:

1. We investigate optimal power control schemes for information transmission and energy sharing in a multi sensor estimation problem with a correlated field of data and minimizing a long-term average distortion cost over an infinite horizon, with centralized causal information and Markovian fading channels and harvested energies.

2. We allow the sensor batteries / energy storage devices to be imperfect and subject to energy leakage.

[2]Temperature measurements across a large tank or humidity measurements across a paper path in a paper mill, which are necessary to guarantee specified quality levels, are examples of such fields.

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3. The optimal stationary power control scheme is found by a stochastic control approach using a Markov decision process (MDP) formulation, where the optimal energy values for information transmission and sharing are found by solving a Bellman dynamic programming equation using *relative value iteration*, see [40].

4. Motivated by practical scenarios, where full statistical information about the harvested energy and fading channel dynamics may not be available, we present a Q-learning algorithm, that yields a suboptimal solution for the power control problem without requiring exact knowledge of all system parameters.

5. We conduct a comparative performance investigation of the optimal solution obtained by using the relative value iteration algorithm with the suboptimal Q-learning algorithm and two simple heuristic policies via suitably chosen numerical examples, illustrating the advantages and disadvantages of each scheme. The benefits of energy sharing, and how the average distortion depends on various parameters such as cross correlation terms, energy transfer efficiency and energy leakage are discussed in detail.

The rest of the paper is organized as follows: The system model is presented in Section 2. Section 3 studies the infinite-horizon optimal power control problem. Three suboptimal power control policies, namely Q-learning and two heuristic policies, are proposed in Sections 4 and 5, respectively. The performances of all considered power control policies are compared by means of numerical examples presented in Section 6, followed by concluding remarks in Section 7.

2. SYSTEM MODEL

We consider a star-network with M sensors and an FC. Each sensor m individually measures a signal of interest $\theta_m(k)$, at discrete-time instants $k \in \{1,2,3,\dots\}$ subject to measurement noise. The measurements are spatially correlated between the sensors. The remote sensors transmit their information to the FC, which estimates the vector $\theta(k) = (\theta_1(k),\theta_2(k),\dots,\theta_M(k))^{\mathrm{T}}$ given the measurements received. We consider an analog amplify and forward uncoded transmission strategy

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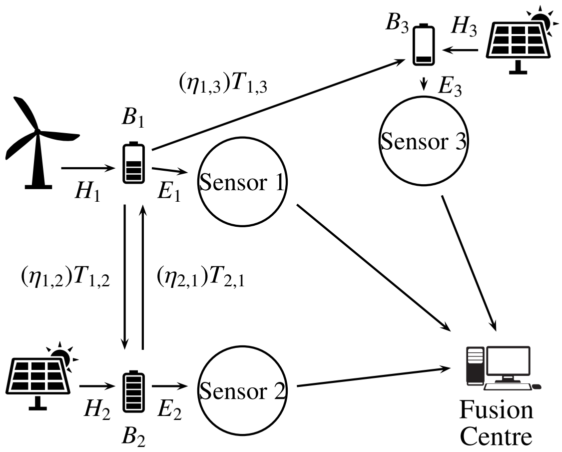

Figure 1. System setting (icons taken from [41])

subject to additive noise, [43]. Each sensor is equipped with a local battery/energy storage device, an energy harvester, and a unit to transmit and receive energy from other sensors, along with a transceiver for information transmission and reception, subject to transmission losses. A scheme showing a simple system with three sensors is depicted in Fig. 1. The description of the individual parts is given below.

2.1. Source Model and Sensor Measurements

We consider $\theta_m(k)$ to be an i.i.d., band-limited Gaussian process with zero mean. The measurements of the sensors are spatially correlated such that its covariance matrix (possibly non-diagonal) is $R_\theta = \mathbb{E}\left\{\theta(k)\theta^{\mathrm{T}}(k)\right\}$. We assume that $R_\theta > 0$ (positive definite). The measurements of sensor m, denoted $x_m(k)$, are subject to measurement noise, $n_m(k)$, such that

$$x_m(k) = \theta_m(k) + n_m(k) \tag{1}$$

for $1 \leq m \leq M$ and $k \geq 1$. The measurement noises $n_m(k)$ are assumed to be i.i.d. Gaussian, mutually independent and also independent of $\theta(k)$. Further, it is assumed that they have zero mean and variances σ_m^2.

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2.2. *Energy Harvester, Energy Sharing and Battery Dynamics*

Each sensor is equipped with an energy harvester to gather energy from the environment. The harvested energy at sensor m at time k, denoted by $H_m(k)$, is described as a first-order homogeneous finite-state irreducible and aperiodic Markov chain, motivated by empirical measurements reported in [42]. We further assume that the Markov chain is unichain, that is, it has a single recurrent class and a possibly empty set of transient states. It is assumed that the harvested energies are mutually independent and independent of the process $\theta(k)$ and the measurement noise. We consider a slotted time model. For simplicity, each time-slot is assumed to be equal to the sampling period between two discrete sampling instants. The energy harvested at time slot k is stored in the battery, and can be used for data transmission to the FC or for energy sharing with neighboring sensors in time slot $k + 1$. The energy used to transmit data from sensor m to the FC at time k is denoted $E_m(k)$. The transmission model will be described in detail in Subsection 2.3.

Each sensor can transmit energy to neighboring sensors and also receive energy from neighboring sensors via directed wireless energy transfer. This can be realized, for instance, by energy transfer between two resonant objects such as discussed in [24, 25], the use of laser beams, or by the use of beamforming radiowaves. The set of neighboring sensors from which sensor m can receive energy is denoted by $\mathcal{N}_{\mathrm{R},m}$ and the set of neighboring sensors to which sensor m can transmit energy is denoted by $\mathcal{N}_{\mathrm{T},m}$. The energy transferred from sensor m to sensor n at time k is denoted by $T_{m,n}(k)$. The efficiency of the energy transfer link from sensor m to sensor n, which accounts for losses in the wireless energy transfer process, is given by $\eta_{m,n} < 1$. In general, the efficiencies $\eta_{m,n}$ can be functions of time, i.e., $\eta_{m,n}(k)$. Unless explicitly mentioned, we will assume time-invariant efficiencies throughout this work.

Further, we assume that during each time interval, some stored energy in the battery is lost due to leakage, [34, 35]. Thus, if no energy is added or used at time k, at time step $k + 1$ only a fraction $\mu \in [0,1]$ of the energy stored in the battery at time k is available for use. Hence, using the notation above, the dynamics of the battery level of sensor m at time $k + 1$ is similar to the model in [36] and

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is given by

$$B_m(k+1) = \min\left\{\left(B_m(k) + H_m(k) - E_m(k) - \sum_{n\in\mathcal{N}_{\mathrm{T},m}} T_{m,n}(k) + \sum_{n\in\mathcal{N}_{\mathrm{R},m}} \eta_{n,m} T_{n,m}(k)\right)\mu; \quad \hat{B}_m\right\}, \tag{2}$$

where $\hat{B}_m$ denotes the maximal battery capacity of sensor m.[3]

2.3. Transmission Model

Each sensor has a transmitter using an analog amplify and forward uncoded strategy.[4] Hence, at each time-slot k, sensor m transmits its measurement $x_m(k)$ amplified by a factor of $\sqrt{\alpha_m(k)}$. The energy needed for transmission is then given by

$$E_m(k) = \alpha_m(k)\left((R_\theta)_{m,m} + \sigma_m^2\right) \tag{3}$$

where $(R_\theta)_{m,n}$ denotes element m,n of matrix R_θ. The channel power gain of the m-th channel between sensor m and the FC, $g_m(k)$, is assumed to be a first-order stationary and homogeneous finite-state Markov block-fading process [44]. We assume that the channel gains are mutually independent and independent of the harvested energies. Similar to the harvested energies, we assume that the Markov chain is unichain. We further assume that within each block, the channel remains constant. For simplicity, the duration of each fading block is assumed to be the same as the duration of each transmission slot. We consider an orthogonal multiple access scheme between the sensors and the FC, which can be implemented for instance via orthogonal frequency division multiple access (OFDMA). The received signal at the FC from sensor m at time k is $z_m(k) = \sqrt{\alpha_m(k) g_m(k)} x_m(k) + \zeta_m(k)$ where $\zeta_m(k)$ is assumed to be i.i.d. additive white Gaussian noise with variance ξ_m^2.

[3]The energy storage model (2) differs from the model in [36] in two ways: (i) the model in [36] does not consider energy transfer, and (ii) the energy harvesting term is not affected by energy leakage. However, by simply rescaling the harvested energy levels accordingly, both models can be converted into each other when the energy transfer terms are ignored.

[4]Optimality of analog transmission for multi sensor estimation of a memoryless Gaussian source over a coherent multiaccess channel was shown in [43]. Further, this scheme is very simple to implement since it does not require complex coding/decoding, and incurs no other delay than propagation delay.

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2.4. Distortion Measure at the Fusion Centre

At the FC, the minimum mean-square error (MMSE) estimator (see [45]) provides the vector of estimates $\hat{\theta}(k) = \left(\hat{\theta}_1(k),\hat{\theta}_2(k),\ldots,\hat{\theta}_M(k)\right)^{\mathrm{T}}$ given the vector of received signals

$$\mathbf{z}(k) = (z_1(k),\ldots,z_M(k))^{\mathrm{T}} = \mathbb{H}\theta(k) + \mathbf{v}(k) \tag{4}$$

with

$$\mathbb{H} = \mathrm{diag}\left(\sqrt{\alpha_1 g_1}, \sqrt{\alpha_2 g_2}, \ldots, \sqrt{\alpha_M g_M}\right) \tag{5}$$

and

$$\mathbf{v} = \left(\sqrt{\alpha_1 g_1}n_1 + \zeta_1, \sqrt{\alpha_2 g_2}n_2 + \zeta_2, \ldots, \sqrt{\alpha_M g_M}n_M + \zeta_M\right)^{\mathrm{T}} \tag{6}$$

(where we dropped the dependence on k for brevity).[5] Then, the distortion measure at the FC is

$$D(\mathbf{E}(k),\mathbf{g}(k)) :=\mathrm{trace}\left(\mathbb{E}\left\{\left(\theta(k)-\hat{\theta}(k)\right)\left(\theta(k)-\hat{\theta}(k)\right)^{\mathrm{T}}\right\}\right) = \mathrm{trace}\left(\left(\mathbb{H}^{\mathrm{T}}R_{\mathrm{v}}^{-1}\mathbb{H} + R_{\theta}^{-1}\right)^{-1}\right) \tag{7}$$

where $\mathbf{E}(k) = (E_1(k), E_2(k), \ldots, E_M(k))$ is the vector of all transmission energies, $\mathbf{g}(k) = (g_1(k), g_2(k), \ldots, g_M(k))$ is the complete vector of all channel gains and $R_{\mathrm{v}} = \mathrm{diag}\left(\alpha_1 g_1 \sigma_1^2 + \xi_1^2,\ldots,\alpha_M g_M \sigma_M^2 + \xi_M^2\right)^{\mathrm{T}}$. Note that the distortion $D(\mathbf{E}(k),\mathbf{g}(k))$ is a random process since $\theta(k)$ is a random variable. Hence, designing optimal predictive power control strategies is a difficult and challenging task.

2.5. Information Patterns

In this paper, we will consider a *causal* information pattern where only information of current and past channel gains and harvested energies is available. In particular, we consider centralized information, where the FC has causal information of all the channel gains, harvested energies and battery levels of all sensors. This can be achieved in practice by the FC transmitting periodic pilot signals to the sensors at the beginning of each transmission slot, from which the sensors estimate their channels and report back their channel gains and previously harvested energies or current battery levels to the FC via orthogonal control channels. We assume the channels between the

[5]It is also assumed that the sensor noise parameters σ_m, and the channel noise variances ξ_m are known at the FC.

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sensors and the FC are reciprocal, such as in a time-division-duplex framework. The FC computes the optimal power control schemes and informs the sensors at each slot.[6]

3. INFINITE-TIME HORIZON OPTIMAL ENERGY ALLOCATION

In this section, we formulate an infinite-time horizon predictive control problem subject to energy constraints (2) to minimize the overall long-term average distortion (7) at the FC. It is considered that only causal information is available. Hence, the unpredictable future wireless fading channel gains and harvested energies are not known a priori and the information available at time $k \geq 1$ is

$$\mathcal{I}_k = \{\mathbf{g}(k), \mathbf{H}(k), \mathbf{B}(k), \mathcal{I}_{k-1}\} \tag{8}$$

where $\mathbf{H}(k) = (H_1(k), H_2(k), \ldots, H_M(k))$ is the vector of harvested energies and $\mathbf{B}(k) = (B_1(k), B_2(k), \ldots, B_M(k))$ is the vector of battery levels at time k, and $\mathcal{I}_1 = \{\mathbf{g}(1), \mathbf{H}(1), \mathbf{B}(1)\}$. The information $\mathcal{I}_k$ is used at each time slot k at the FC to decide the amount of energy used for data transmission from the sensors to the FC, i.e., $E_m(k)$ for all $m = 1,2,\ldots,M$, and the amount of energy transferred between sensors, i.e., $T_{n,m}(k)$ for all $m = 1,2,\ldots,M$ and $n \in \mathcal{N}_{\mathrm{T},m}$. A power control policy is a set of functions to determine $(\{E_m(k)\},\{T_{m,n}(k)\}) : m \in \{1, 2, \ldots, M\}$, and $n \in \mathcal{N}_{\mathrm{T},m}\}$. A policy is feasible if the energy constraints

$$E_m(k) \geq 0, \quad T_{m,n}(k) \geq 0, \quad E_m(k) + \sum_{n \in \mathcal{N}_{\mathrm{T},m}} T_{m,n}(k) \leq B_m(k) \tag{9}$$

are almost surely (a.s.) satisfied for all $1 \leq m,n \leq M$ and $k \geq 1$. The admissible control set is the set of all possible power control policies, which are based only on the causal information set $\mathcal{I}_k$ and do not violate the energy constraints (9). For future reference, we define $\mathbf{T}(k)$ as the matrix with entries $(\mathbf{T}(k))_{m,n} = T_{m,n}(k)$ for $n \in \mathcal{N}_{\mathrm{T},m}$ and $(\mathbf{T}(k))_{m,n} = 0$ otherwise.

[6]The communication overhead between the sensors and the FC for reporting channel gains and battery levels does, of course, also consume energy at the sensors. This is not explicitly taken into account in this work. However, if this energy consumption is constant for each transmission slot, then it can be easily taken into account by subtracting this energy from the maximum battery level and defining a modified maximum battery level for each sensor.

3.1. Infinite-Time Horizon Stochastic Predictive Control Problem

We aim to find the optimal power control scheme that minimizes the expected average distortion measure over an infinite-time horizon. The optimization problem is described as the following stochastic control problem: Find a power control policy, which determines $\mathbf{E}(k)$ and $\mathbf{T}(k)$, such that the following cost function is minimized

$$\limsup_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \mathbb{E}\left\{D(\mathbf{E}(k),\mathbf{g}(k))\right\}, \tag{10}$$

(9) subject to being satisfied a.s. for $1 \leq m,n \leq M$ and $1 \leq k \leq K$, and $B_m(k)$ satisfying (2).

3.2. Stationary Optimal Energy Allocation Policy

The stochastic control problem (10) with centralized information (8) can be regarded as a Markov Decision Process (MDP) formulation $\{\mathcal{S},\mathcal{A},\mathcal{P}\}$ with state space $\mathcal{S} = \{\mathbf{B},\mathbf{g},\mathbf{H}\}$ and action space $\mathcal{A} = \{\mathbf{E},\mathbf{T}\}$. The transition probability from state $\mathcal{S}$ to $\mathcal{S}'$ under action $\mathcal{A}$, i.e., $\mathcal{P}(\mathcal{S}'|\mathcal{S},\mathcal{A})$ can be derived from the battery dynamics (2) while considering the Markov chains describing the channel gains and harvested energies. See [40, 46] for further details.

To simplify notation, the vector of channel gains, harvested energies, battery levels and energy consumption and the matrix of energy shared at time k are denoted $\mathbf{g} = \mathbf{g}(k)$, $\mathbf{H} = \mathbf{H}(k)$, $\mathbf{B} = \mathbf{B}(k)$, $\mathbf{E} = \mathbf{E}(k)$ and $\mathbf{T} = \mathbf{T}(k)$, respectively, and the corresponding vectors of channel gains, harvested energies and battery levels at time $k+1$ are denoted $\tilde{\mathbf{g}} = \mathbf{g}(k+1)$, $\tilde{\mathbf{H}} = \mathbf{H}(k+1)$ and $\tilde{\mathbf{B}} = \mathbf{B}(k+1)$, respectively.

Under the given assumptions, one can show the existence of a stationary optimal power control policy computed offline from a Bellman dynamic programming equation given in Theorem 1 below.

Theorem 1. *Suppose that a unichain power control policy*[7] *exists and consider the average-cost optimality Bellman equation*

$$\rho + V(\mathbf{g},\mathbf{H},\mathbf{B}) = \min_{\mathbf{E},\mathbf{T}} \left\{D(\mathbf{E},\mathbf{g}) + \mathbb{E}\left\{V\left(\tilde{\mathbf{g}},\tilde{\mathbf{H}},\tilde{\mathbf{B}}\middle|\, \mathbf{g},\mathbf{H},\mathbf{E},\mathbf{T}\right)\right\}\right\} \tag{11}$$

[7] A unichain policy is a stationary policy under which the associated Markov chain has a single recurrent class, that is, all states are visited an infinite number of times with probability 1.

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where $\mathbf{E}$ *and* $\mathbf{T}$ *satisfy the energy constraints given in* (9) *and* V *is the relative value function. Then the infinite-time horizon stochastic control problem* (10) *has a unique solution.*

Further, if the set of possible policies includes at least one policy under which energy is used for data transmission or transferred to neighboring nodes, such that the associated Markov chain of battery levels is unichain, then the value of the infinite-time horizon stochastic control problem (10) *is given by* ρ*, which is the unique solution of* (11)*. The optimal average cost* ρ *is independent of the initial conditions* $\mathbf{g}(0)$*,* $\mathbf{H}(0)$ *and* $\mathbf{B}(0)$*.*

Proof

Since it is assumed that the Markov chains of the harvested energies and the channel gains are unichain and that a stationary unichain policy exists, it can be shown that (11) has a unique solution by following similar steps as in [47, Chap. 4.2, Prop. 2.5]. Then, by [47, Chap. 4.2, Prop. 2.6], the solution of (11) is independent of the initial state. □

Remark 1. The stationary optimal solution to the stochastic control problem (10) is given by

$$\{\mathbf{E}^o(\mathbf{g},\mathbf{H},\mathbf{B}),\mathbf{T}^o(\mathbf{g},\mathbf{H},\mathbf{B})\} = \underset{\mathbf{E},\mathbf{T}}{\operatorname{argmin}}\left\{D(\mathbf{E},\mathbf{g}) + \mathbb{E}\left[V(\tilde{\mathbf{g}},\tilde{\mathbf{H}},\tilde{\mathbf{B}})|\mathbf{g},\mathbf{H},\mathbf{E},\mathbf{T}\right]\right\} \tag{12}$$

such that $\mathbf{E}$ and $\mathbf{T}$, which satisfy the energy constraints (9) with battery dynamics (2) for all m, and V constitute the solution to the average cost Bellman equation (11).

Remark 2. If a control policy $\{\mathbf{E}^o,\mathbf{T}^o\}$, a measurable function V, and a constant ρ exist, which solve equations (11) and (12), then the control $\{\mathbf{E}^o,\mathbf{T}^o\}$ is optimal and ρ is the optimal cost

$$\rho = \limsup_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}\left\{D(\mathbf{E}(k),\mathbf{g}(k))\right\}. \tag{13}$$

For any other feasible and causal control policy $\{\mathbf{E},\mathbf{T}\}$, we have

$$\rho \leq \limsup_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}\left\{D(\mathbf{E}(k),\mathbf{g}(k))\right\}. \tag{14}$$

More details can be found in [40].

Remark 3. Since the processes $\mathbf{g}$ and $\mathbf{H}$ are mutually independent (also across sensors) finite state Markov chains, the second right hand term involving the expectation in (11) becomes

$$\int_{\tilde{\mathbf{g}},\tilde{\mathbf{H}}} V\left(\tilde{\mathbf{g}},\tilde{\mathbf{H}},\tilde{\mathbf{B}}\right)\prod_{m=1}^{M}\left(\mathbb{P}(\tilde{g}_m|g_m)\mathbb{P}(\tilde{H}_m|H_m)\right)d\tilde{\mathbf{g}}d\tilde{\mathbf{H}} \tag{15}$$

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where $\mathbb{P}(\mathbf{x}|\mathbf{y})$ is the probability of $\mathbf{x}$ given $\mathbf{y}$.

If the processes $\mathbf{g}$ and $\mathbf{H}$ are i.i.d. over time and across the sensors, then the same term in (11) simplifies to

$$\int_{\tilde{\mathbf{g}},\tilde{\mathbf{H}}} V\left(\tilde{\mathbf{g}}, \tilde{\mathbf{H}}, \tilde{\mathbf{B}}\right) \prod_{m=1}^{M} \left(\mathbb{P}(\tilde{g}_m)\mathbb{P}(\tilde{H}_m)\right) d\tilde{\mathbf{g}} d\tilde{\mathbf{H}}. \tag{16}$$

The Bellman equation (11) can be solved using the relative value iteration algorithm. Details can be found in [40]. In order to facilitate the numerical computation, the Bellman equation (11) is solved by discretizing the state and action space, in particular the battery levels and the power level space. (Recall that the state components involving the fading channels and the harvested energy levels are already assumed to be discrete due to the finite-state Markov chain assumption.) It is expected that the solution of the discretized Bellman equation approaches the solution of the continuous valued Bellman equation as the number of discretization levels grows [48].

Remark 4. Note that the results presented in Theorem 1 can be extended to networks with decentralised information. In such cases, it is assumed that sensors only have access to their own battery level and channel gain. Instead of the actual battery level and channel gains of their neighbors, only statistical information such as the energy harvesting and channel gain models are considered to be known. For more details, see [39].

4. Q-LEARNING

Solving the average-cost optimality Bellman equation (11) requires full knowledge of the underlying transition probability matrix $\mathcal{P}$. In practice, the transition probabilities of the Markov process generating the channel gains and the harvested energies may not be perfectly known. In this case, the optimal power control cannot be determined by solving the Bellman dynamic programming equation presented in the previous section. Hence, finding suboptimal algorithms, which do not rely on the complete knowledge of the underlying system, is an important task. In case the state, $\mathcal{S}$, and action space, $\mathcal{A}$, are discrete or discretized (that is, the channel gains, the harvested energies, the battery levels and the allocated energy usage and energy transfer values belong to finite-discrete

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sets) and the fading channels and harvested energies are independent finite-state Markov chains, the average-cost optimality Bellman equation (11) can be simplified to the Q-Bellman equation [49]

$$Q^*(\mathbf{g},\mathbf{H},\mathbf{B},\mathbf{E},\mathbf{T}) = D(\mathbf{E},\mathbf{g}) + \sum_{\tilde{\mathbf{g}},\tilde{\mathbf{H}},\tilde{\mathbf{B}}} \mathbb{P}(\tilde{\mathbf{g}}|\mathbf{g})\mathbb{P}(\tilde{\mathbf{H}}|\mathbf{H})\mathbb{P}(\tilde{\mathbf{B}}|\mathbf{B},\mathbf{H},\mathbf{E},\mathbf{T}) \min_{\tilde{\mathbf{E}},\tilde{\mathbf{T}}\in A(\tilde{\mathbf{B}})} Q^*(\tilde{\mathbf{g}},\tilde{\mathbf{H}},\tilde{\mathbf{B}},\tilde{\mathbf{E}},\tilde{\mathbf{T}}) \tag{17}$$

where $\tilde{\mathbf{E}}$ or $\tilde{\mathbf{T}}$ are the chosen values for $\mathbf{E}$ or $\mathbf{T}$ at the next time step, respectively, and $A(\tilde{\mathbf{B}})$ is the set of all feasible choices of $\tilde{\mathbf{E}}$ or $\tilde{\mathbf{T}}$ given $\tilde{\mathbf{B}}$. The iterative learning algorithm referred to as Q-learning, approximates the average cost for a given set of states and actions, i.e., Q, by adjusting its value according to the recent observed cost, which is here the distortion D. See also [49] and [50], for more details on the stochastic approximation Q-learning algorithm. Assuming that the probabilities $\mathbb{P}(\tilde{\mathbf{g}}|\mathbf{g})$, $\mathbb{P}(\tilde{\mathbf{H}}|\mathbf{H})$ and $\mathbb{P}(\tilde{\mathbf{B}}|\mathbf{B},\mathbf{H},\mathbf{E},\mathbf{T})$ are unknown we obtain

$$Q_1(\mathbf{g},\mathbf{H},\mathbf{B},\mathbf{E},\mathbf{T}) = 0 \quad \text{for all } \mathbf{g},\mathbf{H},\mathbf{B} \text{ and } \mathbf{E},\mathbf{T} \in A(\mathbf{B}) \tag{18}$$

and for all $k \geq 1$

$$Q_{k+1}(\mathbf{g},\mathbf{H},\mathbf{B},\mathbf{E},\mathbf{T}) = Q_k(\mathbf{g},\mathbf{H},\mathbf{B},\mathbf{E},\mathbf{T}) + \gamma(k)\left(D(\mathbf{E},\mathbf{g}) + \min_{\tilde{\mathbf{E}},\tilde{\mathbf{T}}\in A(\tilde{\mathbf{B}})} Q_k(\tilde{\mathbf{g}},\tilde{\mathbf{H}},\tilde{\mathbf{B}},\tilde{\mathbf{E}},\tilde{\mathbf{T}}) - Q_k(\mathbf{g},\mathbf{H},\mathbf{B},\mathbf{E},\mathbf{T})\right) \tag{19}$$

where now $\{\tilde{\mathbf{g}},\tilde{\mathbf{H}},\tilde{\mathbf{B}},\tilde{\mathbf{E}},\tilde{\mathbf{T}}\}$ is the next state after $\mathbf{g},\mathbf{H},\mathbf{B},\mathbf{E},\mathbf{T}$ when $\mathbf{E},\mathbf{T} \in A(\mathbf{B})$ is selected according to the ϵ-greedy method:

$$\{\mathbf{E},\mathbf{T}\} = \begin{cases} \operatorname{argmin}_{\mathbf{E},\mathbf{T}\in A(\mathbf{B})} Q_k(\mathbf{g},\mathbf{H},\mathbf{B},\mathbf{E},\mathbf{T}) & \text{with prob. } 1-\epsilon \\ \text{chosen randomly } \in A(\mathbf{B}) & \text{with prob. } \epsilon \end{cases} \tag{20}$$

The algorithm in (19) converges to the optimal Q values if the step sizes $\gamma(k)$ for all $k \geq 1$ satisfy $\gamma(k) > 0$, $\sum_k \gamma(k) = \infty$ and $\sum_k \gamma^2(k) < \infty$, [49, 50]. Note that convergence is guaranteed for all $\epsilon > 0$, [49, 50]. If ϵ is large, then the algorithm spends more computational effort in exploring the effect of possible choices of $\mathbf{E}$ and $\mathbf{T}$. A small value of ϵ is usually preferred as it often allows to better exploit the knowledge of which choice of $\mathbf{E}$ and $\mathbf{T}$ leads to the minimal expected cost based on the current Q_k.

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5. HEURISTIC POLICIES

The proposed solutions to find power control policies in the two previous sections, i.e., finding the optimal solution via (11) or solving the iterative learning algorithm (17), require a considerable computational effort. In practice, it is often beneficial to investigate simple policies, that provide suboptimal solutions, but require a reasonable computational effort.

5.1. Heuristic 1: Modified greedy policy

A very simple policy is the greedy policy, where each sensor just uses all available energy to transmit its data to the FC. Hence, $E_m(k) = B_m(k)$ for all m independently of the channel gain or any other states. When implementing this policy, there is a considerable risk of not having any energy available to transmit data from some sensor m to the FC at some time k if no energy has been harvested in the previous step. Thus, the greedy policy is slightly modified such that $E_m(k) = \frac{B_m(k)}{2}$, which ensures that at each time step, some energy is available to transmit data from every sensor to the FC, if the initial battery levels are not zero.

5.2. Heuristic 2: Ad hoc policy

A second heuristic policy was derived for a related but slightly different problem in our recent contribution [39], where instead of a correlated field, all sensors measure the same scalar signal of interest $\theta(k)$. We recapitulate the basic principles next.

Assume a simple system with two sensors, where both agents can share energy between each other and have access to full causal information, such as the maximum battery level, mean channel gains and harvested energies, energy transfer efficiencies as well as current channel gains and battery levels.[8] Aiming to minimise the overall distortion at the FC, leads to the problem described in [39], for which necessary optimality conditions are derived. Those have to be simplified in order to reduce

[8]Note that in case of Markovian channel gains or harvested energies, the mean channel gains $\bar{g}_1$ and $\bar{g}_2$ and the mean harvested energies $\bar{H}_1$ and $\bar{H}_2$ are calculated as the dot product of the channel gain levels or harvested energy levels, respectively, and the corresponding stationary distribution.

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the computational complexity and to require only causal information. The simplified necessary conditions for using energy for data transmission to the FC ($E_1(k) \geq 0$), for storing energy in the battery for future use[9] ($F_1(k) \geq 0$) and for transferring energy to sensor 2 ($T_{1,2}(k) \geq 0$) are as follows:

$$E_1(k) \geq 0 \quad \text{if } g_1(k) \geq \bar{g}_1 \text{ and } g_1(k) \geq \eta_{1,2}\bar{g}_2 \tag{21}$$

$$F_1(k) \geq 0 \quad \text{if } \bar{g}_1 \geq g_1(k) \text{ and } \bar{g}_1 \geq \eta_{1,2}\bar{g}_2 \tag{22}$$

$$T_{1,2}(k) \geq 0 \quad \text{if } \eta_{1,2}\bar{g}_2 \geq g_1(k) \text{ and } \eta_{1,2}\bar{g}_2 \geq \bar{g}_1 \tag{23}$$

In case of unlimited battery capacity, these simplified necessary conditions could be used to allocate the energy at time step k. However, since both batteries have limited capacities, storing all energy at time k or transferring all energy from sensor 1 to sensor 2 at time k might be undesirable despite the necessary conditions (22) or (23) being satisfied because it could lead to preventable battery overflow. Instead of determining the power control policy solely based on the necessary conditions, all three options (data transmission, storage, energy sharing) are prioritized and energy is then allocated accordingly with the aim to minimize battery overflow.

This suggests the following basic rules:

(i) Denote the available pwer, that is available at sensor 1 at timw k by $\bar{B}_1 = B_1(k)$. Then, prioritize the three possible energy usage alternatives, i.e., data transmission $E_1(k)$, storage $F_1(k)$ and energy sharing $T_{1,2}(k)$, by sorting $g_1(k)$, $\bar{g}_1$ and $\eta_{1,2}\bar{g}_2$ from highest to lowest.[10] In case $g_1(k) = \bar{g}_1$ or $g_1(k) = \eta_{1,2}\bar{g}_2$, using energy for data transmission has higher priority than storing energy or transferring it to sensor 2, respectively. In case $\bar{g}_1 = \eta_{1,2}\bar{g}_2$ storing energy has higher priority than transferring it to sensor 2. Then allocate the available energy $\bar{B}_1$ according to these priorities.

(ii) If transmitting data to the FC is the next highest priority, use all remaining energy to transmit data to the FC. (Thus, no energy is allocated to a task with a lower priority.)

[9]That is, $F_1(k)$ is the total amount of energy left in battery 1 for future use.

[10]For instance, if $\bar{g}_1 > g_1(k) > \eta_{1,2}\bar{g}_2$, storing energy has the highest priority followed by data transmission to the FC; and transferring energy to the second sensor has the lowest priority.

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(iii) If storing energy has the next highest priority, energy should be stored. To avoid battery overflow (i.e., energy waste), one should never store more energy than necessary to fill the battery to its maximal capacity minus the mean harvested energy. That is

$$F_1(k) = \min\left\{\max\left\{\hat{B}_1(k) - \bar{H}_1; 0\right\}; \bar{B}_1\right\}.$$

In case there is more energy available in the battery than should be stored, the remaining energy should be used according to the next following priority, that is, following the instructions in (ii) or (iv) and setting $\bar{B}_1 \to \bar{B}_1 - F_1(k)$.

(iv) If transferring energy to sensor 2 has the next highest priority, transfer as much energy to sensor 2 to have its battery full for the next time step. To avoid battery overflow, no more energy should be transferred than the battery capacity minus the mean harvested energy of sensor 2. Therefore, $T_{1,2}(k)$ for $\eta_{1,2} > 0$ is given by

$$T_{1,2}(k) = \min\left\{\max\left\{\left(\hat{B}_2 - B_2(k) + E_2(k) - \bar{H}_2\right)/\eta_{1,2}; 0\right\}; \bar{B}_1\right\}.$$

If $\eta_{1,2} = 0$, then $T_{1,2}(k) = 0$. In case there is more energy in the battery than should be transferred, the remaining energy should be used according to the next following priority, that is, following the instructions in (ii) or (iii) and setting $\bar{B}_1 \to \bar{B}_1 - T_{1,2}(k)$.

The ad hoc heuristic power allocation policy is summarised in the flow chart in Fig. 2.

Remark 5. The necessary conditions reported in [39], which lead to the heuristic algorithm given above, have been derived for a system without battery leakage, that is, with $\mu = 1$. However, when assuming little battery leakage, that is, μ close to 1, it can be expected that the heuristic policy can still be applied.

Remark 6. It should be noted that this heuristic policy favors transmitting data to the FC if the current channel gain is higher than the mean. This policy works well for cases where the overall amount of energy available is low. If only little energy is available, then it is beneficial to minimize the overall distortion by transmitting data whenever the channel gain is better than the mean. In contrast, if a lot of energy is already available due to higher mean harvested energy or higher battery

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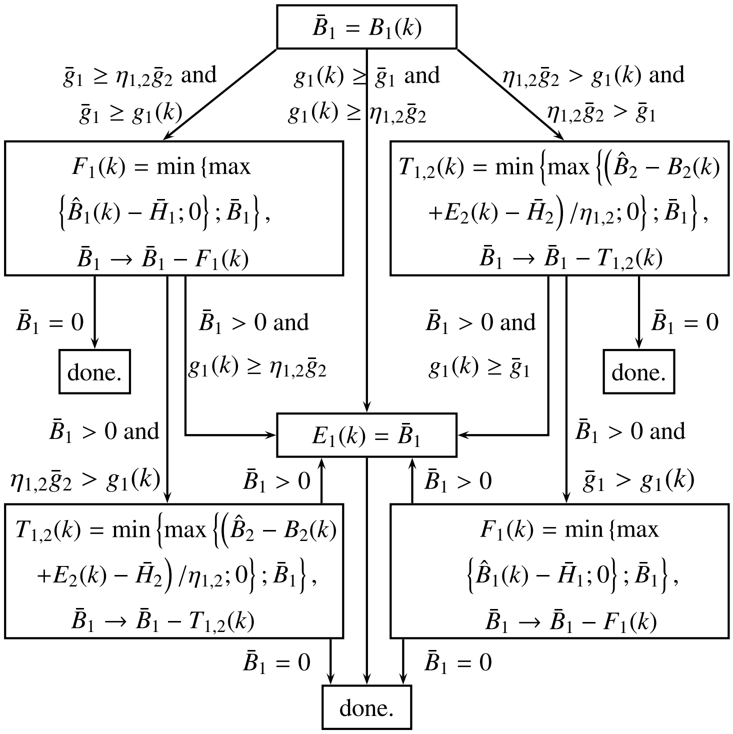

Figure 2. Flow chart of the ad hoc heuristic policy

capacity, then increasing the energy for data transmission further in case of high channel gains leads to a small reduction of the distortion. In these cases it would be better to store energy to be able to transmit data at time steps with poorer channel gains. However, this simple policy cannot distinguish between these two fundamentally different scenarios. It is designed to work well for scenarios with overall little energy availability but its performance may not be as good when higher amounts of energy are available.

6. SIMULATION EXAMPLES

6.1. Numerical Results

In this section, we provide a collection of numerical results that illustrate the performance of the optimal dynamic programming based algorithm, the Q-learning based algorithm and the two heuristic policies against various important parameters such as cross correlation, energy transfer efficiency and battery leakage.

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Example 1 (Effect of Cross Correlation). A system with two sensors is simulated where $\eta_{1,2} = \eta_{2,1} = 0.8$, $\mu = 0$ (no battery leakage), $\hat{B}_1 = \hat{B}_2 = 4mWh$ and $R_\theta = (1, \varphi; \varphi, 1)$, where φ describes the cross correlation between the two measurements θ_1 and θ_2 and is varied between 0 and 0.9.

The fading channel gains and harvested energies are modeled as 3-level discrete Markov chains with the common transition matrix

$$\mathbb{T} = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}. \tag{24}$$

Two cases have been simulated: In the 'balanced scenario', the state space for g_1, g_2 is $\{0,0.5,1\}$ and for H_1 and H_2 is $\{0,1,2\}$. In the 'unbalanced scenario' g_2 and H_1 are 4 times lower than g_1 and H_2, respectively. That is, the state space for g_1 and g_2 are $\{0,0.5,1\}$ and $\{0,0.125,0.25\}$, respectively, while the state spaces of H_1 and H_2 are $\{0,0.5,1\}$ and $\{0,2,4\}$, respectively.

To facilitate the implementation of the dynamic programming algorithm and the Q-learning algorithm, the space for the battery levels and the power levels for data transmission or energy transfer to the neighboring sensor were quantized uniformly using 16 levels. The discretization of the decision variables leads to numerical inaccuracies, which can be addressed by averaging the results over a sufficiently long time span. The Q-learning algorithm was evaluated by the use of two different training time horizons, i.e. 10^4 and 10^6, respectively, and with $\epsilon = 0.1$. After calculating the corresponding Q-values for both training horizons, the performance of the algorithms were evaluated for a given simulation time span by using the Q-values as a look-up table to determine the best choice of $\mathbf{E}$ and $\mathbf{T}$ without adapting Q-values further. Third, the heuristics described in Section 5 were implemented.

The average distortion and the average energy usages for a simulation time span of 10^4 time steps for the optimal solution based on dynamic programming ('DP'), the Q-learning algorithm with the two different training time horizons 10^4 and 10^6 ('Q1' and 'Q2', respectively), and the two heuristics ('h1' and 'h2') are illustrated in the plots in Fig. 3.

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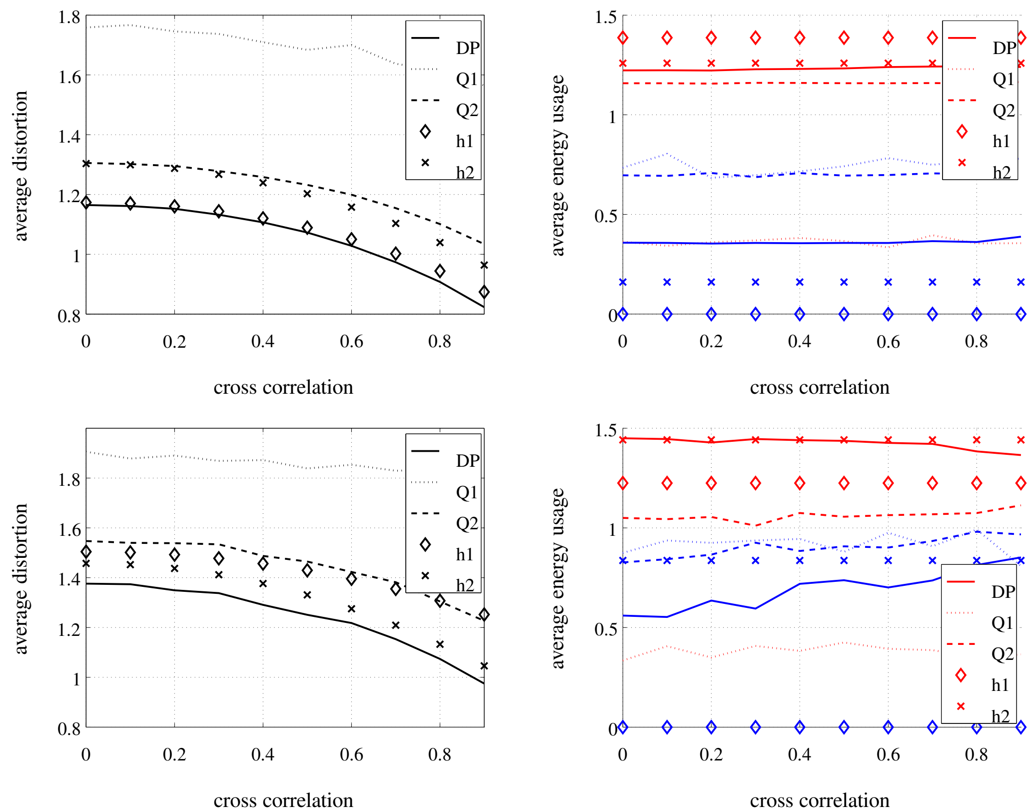

Figure 3. Example 1: average distortion (left) and average energy usage (right, $(E_1 + E_2)/2$ in red, $(T_{1,2} + T_{2,1})/2$ in blue), vs. cross correlation term φ for the 'balanced case' (top) and the 'unbalanced case' (bottom)

It is evident that increasing the cross correlation term φ leads to an overall reduced distortion. As expected, the average distortion is the smallest for the optimal algorithm based on dynamic programming. The performance of the Q-learning algorithm is quite poor if a short training time horizon of 10^4 time steps is used ('Q1'). However, when increasing the training horizon to 10^6 ('Q2') the average distortion is significantly reduced since the optimal policy is better approximated. It is expected that the performance can be further improved using even longer training time horizons. Observe also, that the modified greedy policy ('h1') performs almost as good as the optimal solution ('DP') for the balanced case. In contrast, the ad hoc heuristic ('h2') derived for the related setting in [39] (every sensor measures the same θ) clearly outperforms the modified greedy policy in the unbalanced case.

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Example 2 (Effect of Energy Transfer Efficiency for Low Cross Correlation). The system settings from Example 1 were modified in the following way: Instead of varying the cross correlation term, it is set to $\varphi = 0.2$ while $\eta = \eta_{1,2} = \eta_{2,1}$ is varied between 0 and 1.

The simulations are shown in Figure 4. In the balanced case, the average distortion hardly decreases when increasing the energy transfer efficiency despite the increase of average energy transferred between the sensors. In the unbalanced case, the average distortions obtained for the optimal solution ('DP') and the Q-learning ('Q2') decrease for higher η. As in the previous example, the modified greedy policy ('h1') is more suitable for the balanced case while the ad hoc heuristic ('h2') achieves better results in the unbalanced case. Note that the average distortion for the optimal solution slightly increases between $\eta = 0$ and $\eta = 0.1$. This can be explained by the loss of optimality due to discretization, which is necessary to implement the solution of the Bellman equation on a digital computer resulting in small deviations from the true optimal solution.

In the unbalanced case, it should also be noted that the optimal shared energy increases when the energy transfer efficiency increases from 0 to approximately 0.3. If the energy transfer efficiency is increased further, the optimal amount of energy shared among the sensors remains roughly the same. Since the measurements from the two sensors carry information about two different sources (although correlated) the FC needs to receive data from both sensors in order to estimate both sources. Hence, in the unbalanced case, one sensor needs to share some energy to allow the other sensor to transmit data that can be received at the FC with an acceptable quality. In case wireless energy transfer is possible with a sufficiently high efficiency (such that at least 30% of the transmitted energy is actually received at the receiving sensor), sharing more energy is not beneficial since the other sensor has enough energy already for information transmission with an acceptable distortion level at the FC.

It can also be observed that the curve of the average energy used for data transmission is "bowl shaped": Due to the increase in average shared energy when increasing the energy transfer efficiency from 0 to 0.3, on average, less energy is available for data transmission to the FC. Hence, the average energy usage decreases for low energy transfer efficiencies. However, for higher energy transfer

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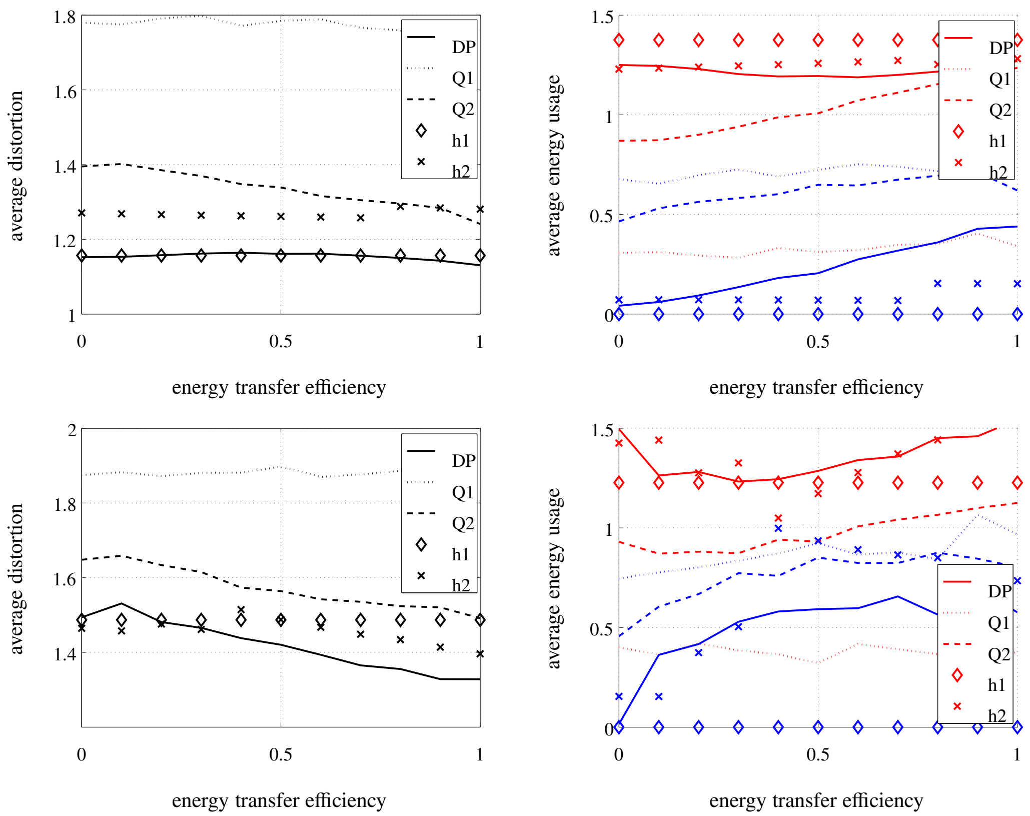

Figure 4. Example 2: distortion (left) and average energy usage (right, $(E_1 + E_2)/2$ in red, $(T_{1,2} + T_{2,1})/2$ in blue), vs. energy transfer efficiency η for the 'balanced case' (top) and the 'unbalanced case' (bottom) for low cross correlation

efficiencies, the amount of shared energy remains almost the same, leading to an increase in average available energy to be used for data transmission.

Example 3 (Effect of Energy Transfer Efficiency for High Cross Correlation). The system settings from Example 2 were solely modified by setting $\varphi = 0.8$.

The simulations in Figure 5 show similar results as in Example 2 for the case of low cross correlation ($\varphi = 0.2$). Due to the higher cross correlation, the average distortion is generally lower in Figure 5 compared to the results in Figure 4. Further, it seems that in case of higher cross correlation, energy transfer offers a larger benefit, since the average energy transferred between the sensors increases more for $\varphi = 0.8$ than for $\varphi = 0.2$.

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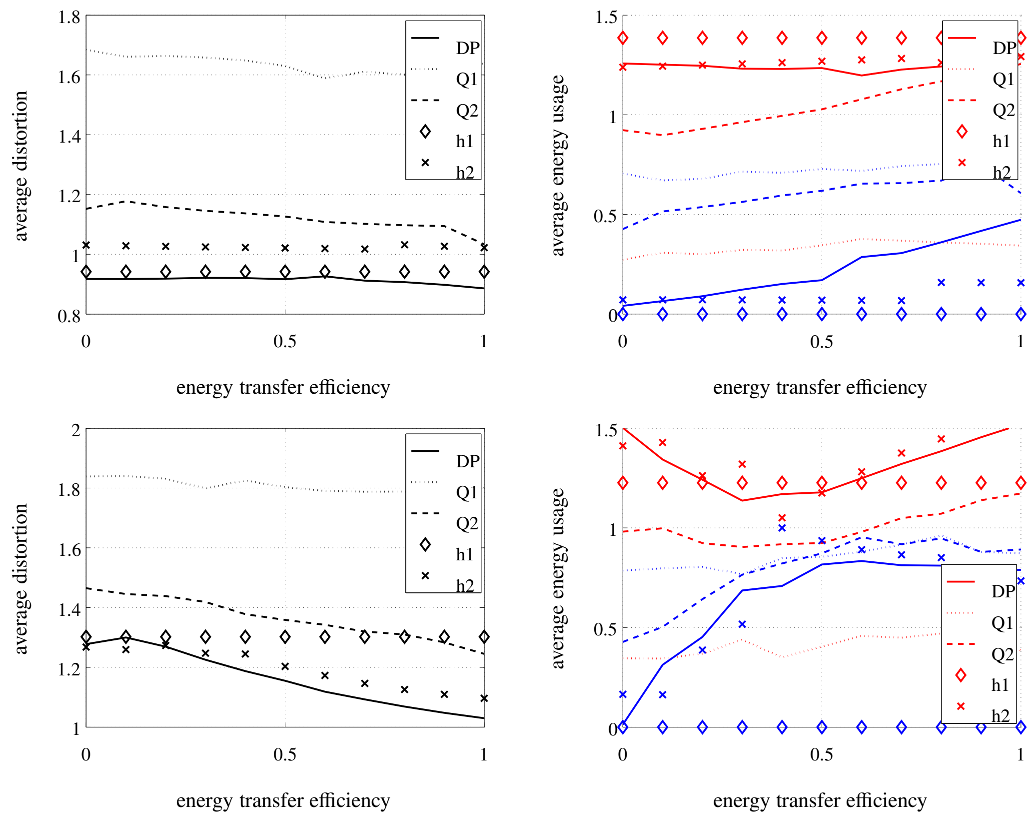

Figure 5. Example 3: distortion (left) and average energy usage (right, $(E_1 + E_2)/2$ in red, $(T_{1,2} + T_{2,1})/2$ in blue), vs. energy transfer efficiency η for the 'balanced case' (top) and the 'unbalanced case' (bottom) for high cross correlation

Example 4 (Effect of Battery Leakage). Here, the system settings are similar to the examples above with setting $\varphi = 0.8$ and $\eta = 0.8$. In contrast to above, the battery leakage parameter μ is varied between 0 (no leakage) to 0.5.

The simulations in Figure 6 show that for all power control policies, in both cases (balanced and unbalanced), a higher battery leakage parameter μ leads to an increase in the average distortion. It is also evident that energy sharing offers more benefits in the unbalanced case compared to the balanced scenario. If the energy loss due to battery leakage increases, then the energy shared among the sensors approaches the average amount of energy used for data transmission. As in the examples above, the modified greedy policy ('h1') is outperformed by the ad hoc policy ('h2') in case of unbalanced networks. In case of balanced networks, the ad hoc heuristic ('h2') outperforms the

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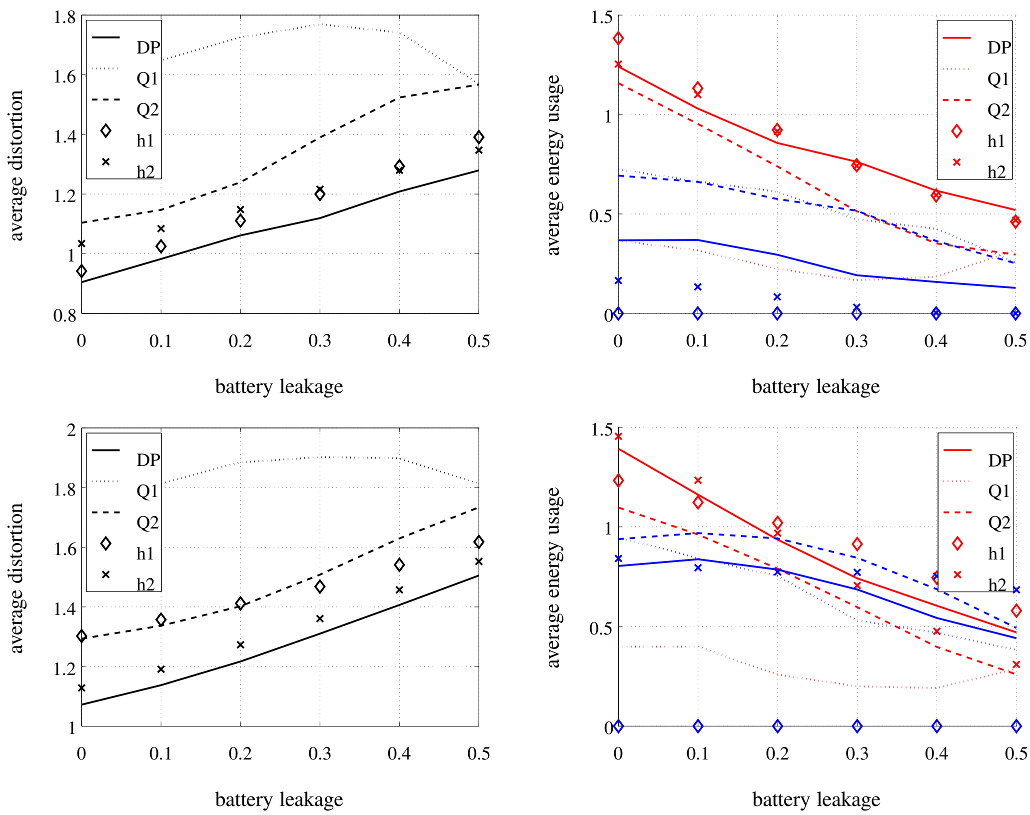

Figure 6. Example 4: distortion (left) and average energy usage (right, $(E_1 + E_2)/2$ in red, $(T_{1,2} + T_{2,1})/2$ in blue), vs. battery leakage factor μ for the 'balanced case' (top) and the 'unbalanced case' (bottom)

modified greedy policy ('h1') for sufficiently high battery leakage. This is despite the ad hoc policy being developed for systems without battery leakage.

6.2. Discussion

When looking at the numerical results above, it becomes clear that the optimal predictive power control scheme outperforms all suboptimal power control algorithms. Also, when considering energy sharing between neighboring sensors by increasing the energy transfer efficiency, the overall distortion decreases, which indicates the usefulness of energy sharing. However, how much the overall distortion can be reduced when implementing the optimal power control solution compared to suboptimal schemes, or when enabling wireless energy transfer, significantly depends on the system settings. For instance, if the system is balanced, i.e., if all sensors have access to similar

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energy harvesting processes and use channels of similar quality, little can be gained from applying the optimal power control or enabling wireless energy transfer. Instead, implementing the very simple modified greedy policy yields almost the same distortions as the optimal solution. In contrast, if the energy harvesting processes and channel gains of the sensors in the system differ significantly, applying the modified greedy policy is not recommendable. Instead, the ad hoc heuristic should be used. It allows wireless energy transfer between neighboring sensors, which appears to be very beneficial in such unbalanced systems.

7. CONCLUSIONS

This paper studied the distortion minimization problem of a multi sensor system, where each sensor transmits its measurement to a FC over a fading channel using uncoded analog forwarding for remote estimation at the FC. The FC computes the optimal predictive power control policy to minimize a long term average distortion cost when using the minimum mean-square error (MMSE) estimator under the following energy constraints: (i) the batteries at the sensors have a limited capacity and are prone to energy leakage, (ii) the sensors can harvest energy from their environment but only causal information about the harvested energies is available, and (iii) the sensors are fitted with transceiver units, that allow them to share energy with their neighbors subject to some loss. Random harvested energies and channel gains are modeled as independent finite-state Markov chains. The FC has causal information about the sensors' channel gains and harvested energy levels.

The optimal solution is obtained via a stochastic predictive control approach resulting in a Bellman dynamic programming equation. A suboptimal Q-learning algorithm, which does not require a priori knowledge of all system parameters, is also studied. Further, to avoid the computational burden of the optimal solution based on dynamic programming techniques, two heuristic ad hoc power control policies are presented. Simulations reveal that the average distortion decreases as the cross correlation and the energy transfer efficiency increase. Further, in most scenarios, the optimal solution (obtained by dynamic programming) clearly outperforms the two sub-optimal policies. It can also be seen that an increase in energy transfer efficiency (for energy

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sharing) and an increase in the cross correlation term have a significantly higher impact on the average distortion if the system is unbalanced, that is, if a sensor has a substantially higher average harvested energy and a poorer channel compared to its neighbor.

The results in this paper reveal important insights into wireless sensor networks with energy harvesting and energy sharing. Despite this paper focusing on relatively simple star networks, the results show that even for those simplistic network settings, the optimal energy allocation policy is far from trivial. Indeed, the findings presented here, form an important base for further investigation in this area as they provide a benchmark for more complicated network topologies. As a next step, more advanced sensor networks should be considered.

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